Optimizations and Extensions for Weighted CFG Parsers

Practical Course in Natural Language Processing

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Outline

Pruning

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- Idea: avoid items that are not part of the best derivation to speed up parsing
- Problem: How can we know these items in advance?
- Practical solution: Use simple methods but take the risk of finding suboptimal derivation.

Pruning for CKY parsing

Consider this slightly modified (red) version of the CKY algorithm:

Require: weighted binary cfg (N, Σ, P, S, μ), word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$
Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid 0 \leq i < j \leq n, A \in N)$ such that, for all i, j, A,
$c_{i,j,A} = \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_j)\} \cup \{0\}$

1: **function** CKY(P, μ, $t_1 \dots t_n$)
2: $(c_{i,j,A} := 0 \mid 0 \leq i < j \leq n, A \in N)$
3: **for** $1 \leq i \leq n$ **do**
4: **for** $A \to t_i \in P$ **do**
5: $c_{i-1,i,A} := \max\{c_{i-1,i,A}, \mu(A \to t_i)\}$
6: **for** $2 \leq r \leq n$ **do**
7: **for** $0 \leq i \leq n - r$ **do**
8: $j := i + r$
9: **for** $m \in \{i + 1, i + 2, \dots, j - 1\}$ **do**
10: **for** $B, C \in N$ **do**
11: **for** $A \in N$ such that $A \to BC \in R$ **do**
12: $c_{i,j,A} := \max\{c_{i,j,A}, \mu(A \to BC) \cdot c_{i,m,B} \cdot c_{m,j,C}\}$
13: **return** c

Pruning for CKY parsing

An abstract "pruning operation" on the table can be included (lines 6, 15).
We speed up the algorithm by skipping the application of cfg rules to table cells with 0 probability (line 12).

Require: weighted binary cfg (N, Σ, P, S, μ), word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$
Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid 0 \leq i < j \leq n, A \in N)$ such that, for all i, j, A, $c_{i,j,A} \leq \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_j)\} \cup \{0\}$

```
1:  function CKY(P, μ, t_1 ... t_n)
2:      (c_{i,j,A} := 0 | 0 ≤ i < j ≤ n, A ∈ N)
3:      for 1 ≤ i ≤ n do
4:          for A → t_i ∈ P do
5:              c_{i-1,i,A} := max{c_{i-1,i,A}, μ(A → t_i)}
6:          (c_{i-1,i,A} | A ∈ N) := prune((c_{i-1,i,A} | A ∈ N))
7:      for 2 ≤ r ≤ n do
8:          for 0 ≤ i ≤ n − r do
9:              j := i + r
10:             for m ∈ {i + 1, i + 2, ..., j − 1} do
11:                 for B, C ∈ N do
12:                     if c_{i,m,B} = 0 or c_{m,j,C} = 0 then continue
13:                     for A ∈ N such that A → BC ∈ R do
14:                         c_{i,j,A} := max{c_{i,j,A}, μ(A → BC) · c_{i,m,B} · c_{m,j,C}}
15:             (c_{i,j,A} | A ∈ N) := prune((c_{i,j,A} | A ∈ N))
16:     return c
```

Pruning for CKY parsing

How can the pruning operation be implemented?

- Threshold beam (set all cell probabilities to 0 if worse than $\theta \cdot$ best cell probability)

Require: family $c = (c_{i,j,A} \in \mathbb{R} \mid A \in N)$, threshold $\theta \in [0, 1]$
Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid A \in N)$

```
1: function PRUNE(c)
2:     m = max_{A∈N} {c_{i,j,A} | A ∈ N}
3:     for A ∈ N do
4:         if c_{i,j,A} < m · θ then
5:             c_{i,j,A} := 0
6:     return c
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  2: $\quad m = \max_{A \in N}\{c_{i,j,A} \mid A \in N\}$
  3: $\quad$ **for** $A \in N$ **do**
  4: $\quad\quad$ **if** $c_{i,j,A} < m \cdot \theta$ **then**
  5: $\quad\quad\quad c_{i,j,A} := 0$
  6: $\quad$ **return** c

- Fixed-sized beam (set all but n best cell probabilities to 0)

  Require: family $c = (c_{i,j,A} \in \mathbb{R} \mid A \in N)$, size $1 \leq n \leq |N|$
  Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid A \in N)$

  1: **function** PRUNE(c)
  2: $\quad [s_1, \ldots, s_n] = n\text{-best}\{c_{i,j,A} \mid A \in N\}$
  3: $\quad$ **for** $A \in N$ **do**
  4: $\quad\quad$ **if** $c_{i,j,A} < s_n$ **then**
  5: $\quad\quad\quad c_{i,j,A} := 0$
  6: $\quad$ **return** c

Pruning for CKY parsing– implementation considerations

- No changes to data structures required.
- More speed-ups might be obtained by not adding items to the table which for sure would later be pruned:
  - Threshold beam: store weight m of currently best item. If new item has weight m' below $\theta \cdot m$, it is save to prune immediately.
  - Fixed-size beam: store weights of the n best items. If the weight of new is below of worst item, prune immediately.

Pruning for deductive parsing

Original algorithm for deductive parsing.

Require: weighted binary cfg (N, Σ, P, S, μ), word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$
Ensure: family $(c_{i,j,A} : \mathbb{R} \mid 0 \le i < j \le n, A \in N)$ such that $c_{i,j,A} = \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_j)\} \cup \{0\}$

function DEDUCE(P, μ, $t_1 \dots t_n$)
  $queue := \{(i-1, A, i, \mu(A \to t_i)) \mid 1 \le i \le n, A \to t_i \in P\}$
  $(c_{i,j,A} := 0 \mid 0 \le i < j \le n, A \in N)$
  while $queue \neq \varnothing$ **do**
    $(i, A, j, w) := \operatorname{argmax}_{(i,A,j,w) \in queue} w$
    $queue \setminus= \{(i, A, j, w)\}$
    if $c_{i,j,A} = 0$ **then**
      $c_{i,j,A} := w$
      $queue \cup= \{(i, A', j', \mu(A' \to AC) \cdot w \cdot c_{j,j',C}) \mid A' \to AC \in P\}$
      $queue \cup= \{(i', A', j, \mu(A' \to BA) \cdot c_{i',i,B} \cdot w) \mid A' \to BA \in P\}$
      $queue \cup= \{(i, A', j, \mu(A' \to A) \cdot w) \mid A' \to A \in P\}$
  return c

Pruning for deductive parsing

Pruning operation on queue is added.

Require: weighted binary cfg (N, Σ, P, S, μ), word $t_1 \dots t_n$ where $t_1, \dots, t_n \in \Sigma$
Ensure: family $(c_{i,j,A} : \mathbb{R} \mid 0 \le i < j \le n, A \in N)$ such that $c_{i,j,A} \le \max\{\mu(d) \mid d \in D_G^A(t_{i+1} \dots t_j)\} \cup \{0\}$

function DEDUCE(P, μ, $t_1 \dots t_n$)
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  while $queue \neq \varnothing$ **do**
    $(i, A, j, w) := \operatorname{argmax}_{(i,A,j,w) \in queue} w$
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    if $c_{i,j,A} = 0$ **then**
      $c_{i,j,A} := w$
      $queue \cup= \{(i, A', j', \mu(A' \to AC) \cdot w \cdot c_{j,j',C}) \mid A' \to AC \in P\}$
      $queue \cup= \{(i', A', j, \mu(A' \to BA) \cdot c_{i',i,B} \cdot w) \mid A' \to BA \in P\}$
      $queue \cup= \{(i, A', j, \mu(A' \to A) \cdot w) \mid A' \to A \in P\}$
      $\mathrm{prune}(queue)$
  return c

Pruning for deductive parsing

Again two options for pruning:

- Threshold beam (remove each queue item if its probability is worse than $\theta \cdot$ prob. of best queue item)

  Require: set $queue \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}$, threshold $\theta \in [0, 1]$
  Ensure: set $queue' \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}$

  1: **function** PRUNE($queue$)
  2: $m = \max_{(i,A,j,w) \in queue} w$
  3: **return** $\{(i, A, j, w) \in queue \mid w > \theta \cdot m\}$

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  2: $m = \max_{(i,A,j,w) \in queue} w$
  3: **return** $\{(i, A, j, w) \in queue \mid w > \theta \cdot m\}$

- Fixed-sized beam (only keep the n most probable queue items)

  Require: set $queue \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}$, size $n \in \mathbb{N}$
  Ensure: set $queue' \subseteq \mathbb{N} \times N \times \mathbb{N} \times \mathbb{R}$

  1: **function** PRUNE($queue$)
  2: $[i_1, \ldots, i_n] = n\text{-best}(queue)$ w.r.t. 4th tuple component
  3: **return** $\{i_1, \ldots, i_n\}$

Pruning for deductive parsing – implementation considerations

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- Beware: Items for small spans are often more probable than items for large spans.
  Risk of pruning "good" large items in favour of "bad" small items.

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  (Solution: see A*-star parsing below)

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A*-parsing

- weighted deductive parsing computes for each item (i, j, A) in the table the weight of the most probable derivation from A to $t_{i+1} \cdots t_j$.

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- A heuristic may also be useful when pruning items during CKY parsing.

A*-parsing – Viterbi outside score

We use the admissible heuristic out:

$$\mathrm{out}(A) = \max_{d \in D_G, u, w \in \Sigma^* : S \overset{d}{\Rightarrow}_G uAw} \mathrm{weight}(d)$$

It can be computed by a variant of the inside/outside algorithm:

function INSIDE
  for $A \in N$ **do**
    $\mathrm{in}(A) := \max(\{\mu(A \to \alpha) \mid A \to \alpha \in R\} \cup \{0\})$
  while not converged **do**
    for $A \in N$ **do**
      $\mathrm{in}(A) = \max(\{\mathrm{in}(A)\} \cup \{\mu(A \to BC) \cdot \mathrm{in}(B) \cdot \mathrm{in}(C) \mid A \to BC \text{ in } R\}$
        $\cup \{\mu(A \to B) \cdot \mathrm{in}(B) \mid A \to B \text{ in } R\})$
function OUTSIDE
  set $\mathrm{out}(B) := \begin{cases} 1 & B = S \\ 0 & \text{otherwise} \end{cases}$ **for each** $B \in N$
  while not converged **do**
    for $B \in N$ **do**
      $\mathrm{out}(B) := \max(\{\mathrm{out}(B)\} \cup \{\mathrm{out}(A) \cdot \mu(A \to BC) \cdot \mathrm{in}(C) \mid A \to BC \text{ in } R\}$
        $\cup \{\mathrm{out}(A) \cdot \mu(A \to CB) \cdot \mathrm{in}(C) \mid A \to CB \text{ in } R\}$
        $\cup \{\mathrm{out}(A) \cdot \mu(A \to B) \mid A \to B \text{ in } R\})$

A*-parsing – parsing algorithm with heuristic

The out-value is now simply included when selecting the best item from the queue:

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  $queue := \{(i-1, A, i, \mu(A \to t_i)) \mid 1 \leq i \leq n, A \to t_i \in P\}$
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  while $queue \neq \varnothing$ **do**
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    if $c_{i,j,A} = 0$ **then**
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      $queue \cup= \{(i', A', j, \mu(A' \to BA) \cdot c_{i',i,B} \cdot w) \mid A' \to BA \in P\}$
      $queue \cup= \{(i, A', j, \mu(A' \to A) \cdot w) \mid A' \to A \in P\}$
  return c

Pruning for deductive parsing – in the context of CKY

Can A*-parsing be applied for CKY parsing?
Yes: in combination with pruning:

- Threshold beam

Require: family $c = (c_{i,j,A} \in \mathbb{R} \mid A \in N)$, threshold $\theta \in [0, 1]$
Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid A \in N)$

1: **function** PRUNE(c)
2: $\quad m = \max_{A \in N}\{c_{i,j,A} \cdot \mathrm{out}(A) \mid A \in N\}$
3: $\quad$ **for** $A \in N$ **do**
4: $\quad\quad$ **if** $c_{i,j,A} \cdot \mathrm{out}(A) < m \cdot \theta$ **then**
5: $\quad\quad\quad c_{i,j,A} := 0$
6: $\quad$ **return** c

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- Fixed-size beam

Require: family $c = (c_{i,j,A} \in \mathbb{R} \mid A \in N)$, size $1 \leq n \leq |N|$
Ensure: family $(c_{i,j,A} \in \mathbb{R} \mid A \in N)$

function PRUNE(c)
  $[s_1, \ldots, s_n] = n\text{-best}\{c_{i,j,A} \cdot \mathrm{out}(A) \mid A \in N\}$
  for $A \in N$ **do**
    if $c_{i,j,A} \cdot \mathrm{out}(A) < s_n$ **then**
      $c_{i,j,A} := 0$
  return c

[Atk+86] M. D. Atkinson et al. “Min-max Heaps and Generalized Priority Queues”. In: *Commun. ACM* 29.10 (Oct. 1986), pp. 996–1000. ISSN: 0001-0782. DOI: 10.1145/6617.6621. URL: http://doi.acm.org/10.1145/6617.6621.

[KM03] Dan Klein and Christopher D. Manning. “A* Parsing: Fast Exact Viterbi Parse Selection”. In: *Proceedings of the 2003 Human Language Technology Conference of the North American Chapter of the Association for Computational Linguistics*. 2003, pp. 119–126. URL: https://www.aclweb.org/anthology/N03-1016.