

# Implementation of PCFG parsers

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# Outline

- 1 Overview
- 2 CKY parsing
- 3 Deductive parsing
- 4 Conclusion

# Overview

## Parser invocation:

```
./pcfg_tool parse grammar.rules grammar.lexicon < sentences
```

## Parser mainloop:

read PCFG  $G$  from files

$G$  ↓  
read sentence  $w$  (1 line) from stdin

$G, w$  ↓  
weighted CKY parsing or deductive parsing

print best derivation tree

$d$  ↓

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# The CKY parsing algorithm (formal notation)

**Require:** CFG  $G = (N, \Sigma, R, S)$  in CNF, sentence  $w = w_1 \dots w_n$  where  $w_1, \dots, w_n \in \Sigma$

**Ensure:** family of sets  $(c_{i,j} \subseteq N \mid 0 \leq i < j \leq n)$  such that  $A \in c_{i,j} \iff D_G^A(w_{i+1} \dots w_j) \neq \emptyset$

```
1: function CKY( $R, w_1 \dots w_n$ )
2:   for  $i := 1$  to  $n$  do
3:      $c_{i-1,i} := \{A \mid A \rightarrow w_i \in R\}$ 
4:   for  $r := 2$  to  $n$  do
5:     for  $i := 0$  to  $n - r$  do
6:        $j := i + r$ 
7:        $c_{i,j} := \{A \mid i < m < j, A \rightarrow BC \in R: B \in c_{i,m}, C \in c_{m,j}\}$ 
8:   return  $(c_{i,j} \mid 0 \leq i < j \leq n)$ 
```

where

- CNF = Chomsky normal form
- $D_G^A(w_i \dots w_j)$  = set of (left) derivations of  $A$  in  $G$  that result in  $w_i \dots w_j$
- $c_{i,j}$  = set of all nonterminals that derive  $w_{i+1} \dots w_j$

# The CKY parsing algorithm (imperative flavor)

**Require:** CFG  $G = (N, \Sigma, R, S)$  in CNF, sentence  $w = w_1 \dots w_n$  where  $w_1, \dots, w_n \in \Sigma$

**Ensure:** family of sets  $(c_{i,j} \subseteq N \mid 0 \leq i < j \leq n)$  such that  $A \in c_{i,j} \iff D_G^A(w_{i+1} \dots w_j) \neq \emptyset$

```
1: function CKY( $R, w_1 \dots w_n$ )
2:    $(c_{i,j} := \emptyset \mid 0 \leq i < j \leq n)$ 
3:   for  $i := 1$  to  $n$  do
4:     for  $A \rightarrow w_i \in R$  do
5:        $c_{i-1,i} := c_{i-1,i} \cup \{A\}$  }  $c_{i-1,i} := \{A \mid A \rightarrow w_i \in R\}$ 
6:   for  $r := 2$  to  $n$  do ▷ length of span
7:     for  $i := 0$  to  $n - r$  do ▷ start of span
8:        $j := i + r$  ▷ end of span
9:       for  $A \in N$  do
10:        for  $m := i + 1$  to  $j - 1$  do ▷ partition position
11:         for  $A \rightarrow BC \in R$  do
12:           if  $B \in c_{i,m}$  and  $C \in c_{m,j}$  then }  $c_{i,j} := \{A \mid i < m < j, A \rightarrow$ 
13:              $c_{i,j} := c_{i,j} \cup \{A\}$  }  $BC \in R: B \in c_{i,m}, C \in c_{m,j}\}$ 
14:   return  $(c_{i,j} \mid 0 \leq i < j \leq n)$ 
```

# From unweighted to weighted CKY parsing

	Unweighted	Weighted
data structure for spans	set of nonterminals $(c_{i,j} \subseteq N \mid 0 \leq i < j \leq n)$	map nonterminals to real numbers $(c_{i,j}: N \rightarrow \mathbb{R} \mid 0 \leq i < j \leq n)$
update operation for rule $r = A \rightarrow \dots$	$c_{i,j} := c_{i,j} \cup \{A\}$	$c_{i,j}(A) := \max\{c_{i,j}(A), p(r)\}$

**Convention:** we will write  $c_{i,j,A}$  rather than  $c_{i,j}(A)$

# The CKY parsing algorithm + weights

**Require:** PCFG  $(N, \Sigma, R, S, p)$  in CNF, sentence  $w = w_1 \dots w_n$  where  $w_1, \dots, w_n \in \Sigma$

**Ensure:** family  $(c_{i,j,A} \in \mathbb{R} \mid 0 \leq i < j \leq n, A \in N)$  such that

$$c_{i,j,A} = \max \left( \{p(d) \mid d \in D_G^A(w_{i+1} \dots w_j)\} \cup \{0\} \right)$$

```
1: function CKY( $R, p, w_1 \dots w_n$ )
2:   ( $c_{i,j,A} := 0 \mid 0 \leq i < j \leq n, A \in N$ )
3:   for  $i := 1$  to  $n$  do
4:     for  $A \rightarrow w_i \in R$  do
5:        $c_{i-1,i,A} := \max\{c_{i-1,i,A}, p(A \rightarrow w_i)\}$ 
6:     for  $r := 2$  to  $n$  do
7:       for  $i := 0$  to  $n - r$  do
8:          $j := i + r$ 
9:         for  $A \in N$  do
10:          for  $m := i + 1$  to  $j - 1$  do
11:            for  $A \rightarrow BC \in R$  do
12:               $c_{i,j,A} := \max\{c_{i,j,A}, p(A \rightarrow BC) \cdot c_{i,m,B} \cdot c_{m,j,C}\}$ 
13:   return ( $c_{i,j,A} \mid 0 \leq i < j \leq n, A \in N$ )
```



# Adding chain rules

Types of rules accepted by classic CKY:

$$A \rightarrow w$$

(with  $A, B, C \in N$  and  $w \in \Sigma$ )

$$A \rightarrow BC$$

What we also want:

$$A \rightarrow B$$

(with  $A, B \in N$ )

Solution: two-phase procedure per cell  $c_{i,j}$

- 1 perform classic CKY step
- 2 compute closure under unary rules

# The CKY parsing algorithm + weights + chain rules

**Require:** binary PCFG  $(N, \Sigma, R, S, p)$ , sentence  $w = w_1 \dots w_n$  where  $w_1, \dots, w_n \in \Sigma$

**Ensure:** family  $(c_{i,j,A} \in \mathbb{R} \mid 0 \leq i < j \leq n, A \in N)$  such that

$$c_{i,j,A} = \max (\{p(d) \mid d \in D_G^A(w_{i+1} \dots w_j)\} \cup \{0\})$$

- 1: **function** CKY( $R, \mu, w_1 \dots w_n$ )
- 2:      $(c_{i,j,A} := 0 \mid 0 \leq i < j \leq n, A \in N)$
- 3:     **for**  $i := 1$  **to**  $n$  **do**
- 4:         **for**  $A \rightarrow w_i \in R$  **do**
- 5:              $c_{i-1,i,A} := p(A \rightarrow w_i)$
- 6:              $(c_{i-1,i,A} \mid A \in N) = \text{UNARY\_CLOSURE}(R, p, (c_{i-1,i,A} \mid A \in N))$
- 7:         **for**  $r := 2$  **to**  $n$  **do**
- 8:             **for**  $i := 0$  **to**  $n - r$  **do**
- 9:                  $j := i + r$
- 10:                 **for**  $A \in N$  **do**
- 11:                     **for**  $m := i + 1$  **to**  $j - 1$  **do**
- 12:                         **for**  $A \rightarrow BC \in R$  **do**
- 13:                              $c_{i,j,A} := \max \{c_{i,j,A}, p(A \rightarrow BC) \cdot c_{i,m,B} \cdot c_{m,j,C}\}$
- 14:                      $(c_{i,j,A} \mid A \in N) = \text{UNARY\_CLOSURE}(R, p, (c_{i,j,A} \mid A \in N))$
- 15:     **return**  $(c_{i,j,A} \mid 0 \leq i < j \leq n, A \in N)$

# The CKY parsing algorithm + weights + chain rules

```
16: function UNARY_CLOSURE( $R, p, (c_A \mid A \in N)$ )
17:    $queue := \{(A, c_A) \mid A \in N, c_A \neq 0\}$ 
18:    $(c_A := 0 \mid A \in N)$ 
19:   while  $queue \neq \emptyset$  do
20:      $(B, q) := \operatorname{argmax}_{(\hat{B}, \hat{q}) \in queue} \hat{q}$ 
21:      $queue := queue \setminus \{(B, q)\}$ 
22:     if  $c_B < q$  then
23:        $c_B := q$ 
24:       for  $A \rightarrow B \in R$  do
25:          $queue := queue \cup \{(A, p(A \rightarrow B) \cdot q)\}$ 
26:   return  $(c_A \mid A \in N)$ 
```

## About backtraces

**What we have:** best weight  $c_{i,j,A}$  for derivations in  $D_G^A(w_{i+1} \dots w_j)$  for each  $A \in N$ ,  $0 \leq i < j \leq n$

**What we want:** the best derivation of  $S$  that results in  $w = w_1 \dots w_n$

During the CKY algorithm:

- store *backtraces* (indicators *how* a weight was computed)
- for best derivation: at most one backtrace per span and nonterminal
- update when weight is updated

After the CKY algorithm:

- recursively read trees from backtraces:

**Require:** family of backtraces  $b$ , each of:  $\perp$ , or  $A \rightarrow t$ , or  $(A \rightarrow B, i, j)$  or  $(A \rightarrow BC, i, m, m, j)$

- 1: **function** BEST\_TREE( $(b_{i,j,A} \mid 0 \leq i < j \leq n, A \in N), i, j, A$ )
- 2:   **if**  $b_{i,j,A}$  of:  $A \rightarrow t$  **then return**  $A \rightarrow t$
- 3:   **else if**  $b_{i,j,A}$  of:  $(A \rightarrow B, i, j)$  **then return**  $(A \rightarrow B)(\text{BEST\_TREE}(b, i, j, B))$
- 4:   **else if**  $b_{i,j,A}$  of:  $(A \rightarrow BC, i, m, m, j)$  **then**
- 5:       **return**  $(A \rightarrow BC)(\text{BEST\_TREE}(b, i, m, B), \text{BEST\_TREE}(b, m, j, C))$

# Let's talk about data structures

- access to grammar rules depends on loops:
  - access by first nonterminal on rhs
  - for some, that be no concern

```
Map<Nt, (Rule, Wt)>  
Set<(Rule, Wt)>
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- weights for each span and nonterminal:
  - usually in a  $(\frac{|w| \cdot (|w| + 1)}{2} \cdot |N|)$ -dimensional vector (dense)
  - or hashmap (sparse)

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Vec<Wt>

Map<(Int, Int, Nt), Wt>

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- or hashmap (sparse)

Vec<Wt>

Map<(Int, Int, Nt), Wt>

- storing backtraces:

- each backtrace: rule and spans and nonterminals (on rule's right-hand side)

Bt = Bin(Rule, [Int; 4])+ Chain(Rule, [Int; 2])+ Term(Rule)

- one backtrace for each span and nonterminal (dense)
- or in a hashmap (sparse)
- or do not store them at all

Vec<Bt>

Map<(Int, Int, Nt), Bt>

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# Deduction systems [Ned03]

- rule-based system (derivation of *items*)
- derive consequence ( $c$ ) from antecedents ( $a_1, \dots, a_k$ )  
for some  $k \in \mathbb{N}$

$$\frac{a_1, \dots, a_k}{c}$$

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for some  $k \in \mathbb{N}$
- compute weight of consequence using weight of  
antecedents ( $w_1, \dots, w_k$ )

$$\frac{a_1 : w_1, \dots, a_k : w_k}{c : f(w_1, \dots, w_k)}$$

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for some  $k \in \mathbb{N}$
- compute weight of consequence using weight of  
antecedents ( $w_1, \dots, w_k$ )
- side condition  $b$

$$\frac{a_1 : w_1, \dots, a_k : w_k}{c : f(w_1, \dots, w_k)} b$$

## Deduction system for parsing weighted cfg [Ned03]

- item  $(i, A, j)$  for each nonterminal  $A$  spanning  $w_{i+1} \dots w_j$

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- combine items  $\frac{(i_0, B_1, i_1): w_1, (i_1, B_2, i_2): w_2, \dots, (i_{k-1}, B_k, i_k): w_k}{(i_0, A, i_k): p(A \rightarrow B_1 \dots B_k) \cdot w_1 \dots w_k} A \rightarrow B_1 \dots B_k \in R$

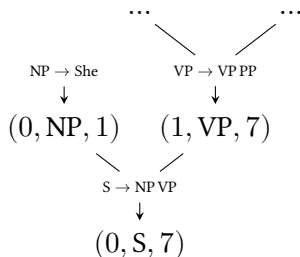
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- goal item:  $(0, S, |w|)$

- deduction system  $\rightsquigarrow$  weighted hypergraph
  - edge from antecedents to consequence
  - can be explored with respect weight
  - hyperpaths to goal item correspond to parse trees





# Weighted deductive parsing algorithm

**Require:** weighted binary cfg  $(N, \Sigma, R, S, p)$ , word  $w_1 \dots w_n$  where  $w_1, \dots, w_n \in \Sigma$

**Ensure:** family  $(c_{i,j,A} \in \mathbb{R} \mid 0 \leq i < j \leq n, A \in N)$  such that

$$c_{i,j,A} = \max (\{p(d) \mid d \in D_G^A(w_i \dots w_j)\} \cup \{0\})$$

- 1: **function** DEDUCE( $R, p, w_1 \dots w_n$ )
- 2:      $queue := \{(i-1, A, i, p(A \rightarrow w_i)) \mid 1 \leq i \leq n, A \rightarrow w_i \in R\}$
- 3:      $(c_{i,j,A} := 0 \mid 0 \leq i < j \leq n, A \in N)$
- 4:     **while**  $queue \neq \emptyset$  **do**
- 5:          $(i, A, j, q) := \operatorname{argmax}_{(\hat{i}, \hat{A}, \hat{j}, \hat{q}) \in queue} \hat{q}$
- 6:          $queue := queue \setminus \{(i, A, j, q)\}$
- 7:         **if**  $c_{i,j,A} = 0$  **then**
- 8:              $c_{i,j,A} := q$
- 9:              $queue := queue \cup \{(i, A', j', p(A' \rightarrow AC) \cdot q \cdot c_{j,j',C}) \mid j < j' \leq n, A' \rightarrow AC \in R\}$
- 10:              $queue := queue \cup \{(i', A', j, p(A' \rightarrow BA) \cdot c_{i',i,B} \cdot q) \mid 0 \leq i' < i, A' \rightarrow BA \in R\}$
- 11:              $queue := queue \cup \{(i, A', j, p(A' \rightarrow A) \cdot q) \mid A' \rightarrow A \in R\}$
- 12:     **return**  $(c_{i,j,A} \mid 0 \leq i < j \leq n, A \in N)$

## Let's talk about data structures ... again

- access of grammar from each rhs nonterminal

Map<Nt, (Rule, Wt)>

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```
Map<Nt, (Rule, Wt)>  
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# Let's talk about data structures ... again

- access of grammar from each rhs nonterminal
- each item may need to store a backtrace
- storing the found items and their weights:

- access from left
- access from right

Map<Nt, (Rule, Wt)>

(Int, Nt, Int, Wt, Bt)

Map<(Int, Nt), Set<(Int, Nt, Int, Wt)>>

Map<(Nt, Int), Set<(Int, Nt, Int, Wt)>>

# Let's talk about data structures ... again

- access of grammar from each rhs nonterminal `Map<Nt, (Rule, Wt)>`
- each item may need to store a backtrace `(Int, Nt, Int, Wt, Bt)`
- storing the found items and their weights:
  - access from left `Map<(Int, Nt), Set<(Int, Nt, Int, Wt)>>`
  - access from right `Map<(Nt, Int), Set<(Int, Nt, Int, Wt)>>`
- storing backtraces:
  - store applied rule and antecedent items  
`Bt = Bin(Rule, [Int; 4])+ Chain(Rule, [Int; 2])+ Term(Rule)`
  - one backtrace for each item `Map<(Int, Nt, Int), Bt>`
  - or do not store them at all

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# General Comments and Tips

- order of loops in CKY algorithm doesn't matter that much, *but*<sup>1</sup>:
  - may be used to cache-optimize,
  - may lead to other optimizations

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<sup>1</sup>Bodenstab [Bod09] discusses this in detail.

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- try to think about efficient access in your data structures
  - don't search in lists
  - indexed access: maps
  - check if you *really* need sets/maps
  - flat data structures are faster than stacked heap allocations

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  - flat data structures are faster than stacked heap allocations
- try not to over-engineer it

---

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