Regular tree grammars

Exercise 6.1. A context-free grammar $G$ (in the standard notation) contains the following rules ($Z$ is the initial nonterminal):

$$
Z \to AZ \mid A \quad (\rho_1 \mid \rho_2)
$$

$$
A \to AbA \mid a \quad (\rho_3 \mid \rho_4)
$$

a) Recall (i.e., specify) the CFG-algebra $CFG$ over \{a, b\}.

b) Specify the RTG $H$ such that $L(G) = \llbracket L(H) \rrbracket_{CFG}$

c) We let $w = ababaa$. Specify a derivation of $G$ recognizing $w$ and show the corresponding tree $\xi \in L(H)$ with $\llbracket \xi \rrbracket_{CFG} = w$.

Exercise 6.2. (based on [Kallmeyer, 2010] Ex. 6.3, Ex. 6.1) What are the languages generated by the following LCFRS over $\Delta$?

1. $G_1 = (N, \Sigma, Z, R)$ where
   - $\Delta = \{a, b, c, d\}$,
   - $N = \{Z, A, B\}$,
   - $\Sigma = \{\langle x_1^{(1)} x_2^{(1)} \rangle, \langle ax_1^{(1)} b \rangle, \langle cx_1^{(1)} d \rangle, \langle \epsilon, \epsilon \rangle\}$, and
   - $R$ contains the rules
     $$
     Z \to \langle x_1^{(1)} x_2^{(1)} x_2^{(2)} \rangle (A, B) \ , \\
     \quad A \to \langle ax_1^{(1)} b \rangle (A) \ , \\
     \quad A \to \langle \epsilon, \epsilon \rangle \ , \\
     \quad B \to \langle cx_1^{(1)} d \rangle (B) \ , \\
     \quad \text{and} \\
     B \to \langle \epsilon, \epsilon \rangle .
     $$

2. $G_2 = (N, \Sigma, S, R)$ where
   - $\Delta = \{a, b\}$,
   - $N = \{S, A\}$,
   - $\Sigma = \{\langle x_1^{(1)} x_2^{(1)} \rangle, \langle ax_1^{(1)} b \rangle, \langle x_2^{(1)} x_1^{(1)} \rangle, \langle \epsilon, \epsilon \rangle\}$, and
• $R$ contains the rules

\[
S \rightarrow \langle x_1^{(1)} x_2^{(1)} \rangle (A), \\
A \rightarrow \langle ax_1^{(1)}, bx_2^{(1)} \rangle (A), \\
A \rightarrow \langle x_2^{(1)}, x_1^{(1)} \rangle (A), \\
A \rightarrow \langle \epsilon, \epsilon \rangle
\]

and

Give a LCFRS that generates the language $L = \{a^k b^k c^k d^k \mid k \geq 0\}$. What fan-out does a LCFRS need to have at least in order to generate $L$?

Literatur

Kallmeyer, L. (2010). *Parsing beyond context-free grammars*. Springer, DOI: 10.1007/978-3-642-14846-0