

Formale Baumsprachen

Task 18 (concatenation and Kleene star for recognizable tree languages)

Let Σ be a ranked alphabet.

- (a) Show that $\text{REC}(\Sigma)$ is closed under top concatenation without using the fact that it is closed under tree concatenation.
- (b) Why can we not use the closure of $\text{REC}(\Sigma)$ under tree concatenation to prove the closure under Kleene star?

Prove or refute the following two statements:

- (c) For every $\alpha \in \Sigma^{(0)}$, the binary operation \cdot_α is associative. Assume that \cdot_α distributes over \cup .
- (d) $(L_1 \cdot_\alpha L_2) \cdot_\beta L_3 = L_1 \cdot_\alpha (L_2 \cdot_\beta L_3)$ for arbitrary $L_1, L_2, L_3 \in \text{REC}(\Sigma)$ and $\alpha, \beta \in \Sigma^{(0)}$.

Let $\Delta = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet.

- (e) Using the construction from the lecture, show that $\{\sigma(\alpha, \beta)\}_\beta^* \cdot_\beta \{\alpha\} \in \text{REC}(\Sigma)$.