Formale Baumsprachen

Task 16 (closure of REC *under intersection, union, and complement)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the following recognizable tree languages

$$L_1 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathrm{pos}(\xi)\colon w \in \{2\}^* \text{ if and only if } \xi(w) \in \{\sigma, \alpha\}\} \text{ and}$$
$$L_2 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathrm{pos}(\xi)\colon \xi(w) = \alpha \text{ only if } |w| \equiv 0 \pmod 2\}.$$

Find finite representations for the following languages:

(a) L_1 (b) L_2 (c) $L_1 \cup L_2$ (d) $L_1 \cap L_2$ (e) $T_\Sigma \setminus L_1$

Task 17 (string automata II)

Let $\Sigma = \{\mathrm{a}, \mathrm{b}\}$ be an alphabet.

(a) Give a finite state automaton $\mathcal{A} = (Q, \Sigma, q_0, F)$ that recognizes

$$L = \{w \in \Sigma^* \mid |w|_\mathrm{a} - |w|_\mathrm{b} \bmod 2 \equiv 0\}.$$

(b) Describe L using a homomorphism between the free monoid $(\Sigma^*, \circ, \varepsilon)$ and the monoid $(\{0,1\}^{Q\times Q}, \times, 1_{Q\times Q})$.

(c) Describe L using a monoid with carrier $(\Sigma^*)^{Q\times Q}$.

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