Formale Baumsprachen

Task 8 (string automata)

Recall the concept of string automata. Let Σ be an alphabet and $\# \notin \Sigma$. We define the ranked alphabet $\Sigma_\# = \Sigma_\#^{(0)} \cup \Sigma_\#^{(1)}$ where $\Sigma_\#^{(0)} = \{\#\}$ and $\Sigma_\#^{(1)} = \Sigma$. Moreover, we define the $\Sigma_\#$-algebra (Σ^*, θ) where $\theta(\#) = \varepsilon$ and $\theta(a)(w) = wa$ for every $a \in \Sigma$ and $w \in \Sigma^*$.

(a) Show that Σ^* is initial in the class of $\Sigma_\#$-algebras.

(b) We consider $\Sigma = \{a, b\}$ and the language $L = \{a^n b^m \mid n, m \in \mathbb{N}\}$. Sketch the diagram of a total deterministic finite-state automaton accepting L and model the transition table using a finite $\Sigma_\#$-algebra Q. How can we interpret the uniquely determined homomorphism $h\colon \Sigma^* \to Q$?

(c) Convince yourself that any total deterministic finite-state automaton can be modeled as a quadruple $\mathcal{A} = (Q, \Sigma, \delta, F)$ where (Q, δ) is a finite $\Sigma_\#$-algebra and $F \subseteq Q$. Define the language accepted by $\mathcal{A}$ using the homomorphism $h\colon \Sigma^* \to Q$.

Task 9 (non-deterministic bottom-up tree automata)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \gamma^{(0)}\}$. Give a nondeterministic bu-ta which accepts exactly the language of all $\xi \in \mathrm{T}_\Sigma$ containing a β-leaf somewhere between an α- and a γ-leaf, reading leaves left-to-right or right-to-left. Try to use as few states and transitions as possible.

Task 10 (non-deterministic top-down tree automata)

For each of the following tree languages, give a td-ta which accepts exactly that language. Which of these languages can be accepted by some deterministic td-ta?

(a) $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $L = \{\xi \in \mathrm{T}_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$.

(b) $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $L = \{\xi \in \mathrm{T}_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$.

(c) $\Sigma = \{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}, \epsilon^{(0)}\}$ and
$L = \{\xi \in \mathrm{T}_\Sigma \mid \xi \text{ contains an } \alpha \text{ somewhere above a } \beta \text{ or a } \beta \text{ somewhere above a } \gamma\}$.

`http://www.orchid.inf.tu-dresden.de` `Tobias.Denkinger@tu-dresden.de`