

Formale Baumsprachen

Task 4 (definition by structural induction)

Let Σ be a ranked alphabet, X be a set, $\xi, \xi_1, \dots, \xi_k \in T_\Sigma$, and $\zeta \in T_\Sigma(X_k)$. Define the following functions by structural induction:

- (a) $\text{yield}(\xi)$, the sequence of leaves in ξ from left to right; and
- (b) $\zeta[\xi_1, \dots, \xi_k]$, the tree obtained from ζ by replacing every occurrence of x_i by ξ_i for every $i \in \{1, \dots, k\}$.

Task 5 (proof by structural induction)

Let Σ be a ranked alphabet, X be a set, $\xi, \zeta \in T_\Sigma(X)$, and $w \in \text{pos}(\xi)$. Prove or refute the following statements:

- (a) $\text{height}(\xi) = 1 + \max \{|w| \mid w \in \text{pos}(\xi)\}$.
- (b) $|\text{pos}(\xi)| = |\text{sub}(\xi)|$.
- (c) $\text{pos}(\xi|_w) \subseteq \text{pos}(\xi)$.
- (d) $\text{sub}(\xi|_w) \subseteq \text{sub}(\xi)$.
- (e) $\text{size}(\xi[\zeta]_w) = \text{size}(\xi) + \text{size}(\zeta) - \text{size}(\xi|_w)$.