Formale Baumsprachen

Task 1 (finite state (string) automata)

Let $\Sigma = \{\mathrm{a}, \mathrm{b}\}$ be an alphabet (without ranks).

(a) Give a finite state automaton $\mathcal{A} = (Q, \Sigma, q_0, F, T)$ that recognizes

$$L = \{w \in \Sigma^* \mid |w|_\mathrm{a} - |w|_\mathrm{b} \bmod 2 \equiv 0\}.$$

(b) Draw a graph representing $\mathcal{A}$.

(c) Give a run $\theta \in T^*$ of $\mathcal{A}$ on aaba.

Task 2 (ranked alphabets)

(a) Extend the intersection, union, and subset relation to ranked alphabets.

(b) Define minimal ranked alphabets Δ_1 and Δ_2 such that $\xi_1 \in \mathrm{T}_{\Delta_1}$ and $\xi_2 \in \mathrm{T}_{\Delta_2}$.

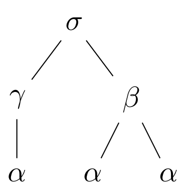
(c) Prove or refute: There is a ranked alphabet Γ such that $\xi_1, \xi_2 \in \mathrm{T}_\Gamma$.

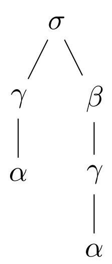
Task 3 (tree domain)

In the lecture we defined trees as well-formed expressions. An alternative definition characterises a tree as a tuple (t, φ) where, intuitively, t is a set of *Gorn addresses* that is closed under certain operations and φ assigns a symbol from some alphabet Δ to every element of t.

(a) Give a formal definition of trees over Δ in the above sense.

(b) Consider the following trees:

$\xi_1 =$ 

$\xi_2 =$ 

Give the corresponding tuples (t_1, φ_1) and (t_2, φ_2) according to the definition in Task 3 (a).

(c) Formally define the following characteristics of trees in the sense of Task 3 (a):

(i) height (i.e. maximal length of a path)

(ii) size (i.e. number of nodes)

(iii) set of positions (i.e. set of addresses to all nodes)

(iv) set of subtrees

(v) label at a position

(vi) subtree at a position

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