Formale Baumsprachen

Task 25 (monadic second-order logic on trees)

(a) Define macros for conjunction, implication, equivalence, first and second order universal quantification, an edge relation, a leaf predicate, and a root predicate.

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the MSO-formula

$$\varphi = \exists U.\neg\exists x.\exists y.\mathrm{edge}_2(x, y) \wedge \mathrm{label}_\sigma(y) \wedge x \in U$$

over Σ where $x, y \in \mathcal{V}_1$ and $U \in \mathcal{V}_2$.

(b) Calculate $\mathrm{Free}(\varphi)$ and $\mathrm{Bound}(\varphi)$ using the definitions from the lecture.

(c) Is φ closed?

Consider the tree $\xi = \sigma(\gamma(\alpha), \beta)$ and the following functions:

$$\begin{aligned}
\rho_1 &: \; x \mapsto \varepsilon, \;\; x' \mapsto 1, \;\; y \mapsto 11, \;\; y' \mapsto 2, \\
\rho_2 &: \; x \mapsto \varepsilon, \;\; x' \mapsto \varepsilon, \;\; \bar{x} \mapsto 1, \;\; y \mapsto 11, \;\; y' \mapsto 2, \\
\rho_3 &: \; X \mapsto \{\varepsilon, 1\}, \;\; Y \mapsto \{11, 2\}, \text{ and} \\
\rho_4 &: \; X \mapsto \emptyset, \;\; Y \mapsto \{1, 2, 3\}, \;\; x \mapsto \varepsilon.
\end{aligned}$$

(d) Which of the functions $\rho_1, ..., \rho_4$ are assignments for ξ? Give the appropriate sets of variables.

(e) Encode the assignments from Task 25 (d) as trees.

(f) Which of the trees obtained in Task 25 (e) are valid?

Let $\mathcal{V} = \{x, y, U\}$. Construct MSO-formulas φ_1, φ_2, and φ_3 such that $\mathrm{Free}(\varphi_1) \cup \mathrm{Free}(\varphi_2) \cup \mathrm{Free}(\varphi_3) \subseteq \mathcal{V}$ and for every $\xi \in T_\Sigma$ and $\mathcal{V}$-assignment ρ for ξ:

(g) $(\xi, \rho) \models \varphi_1$ iff there is a downward path from the node $\rho(x)$ to the node $\rho(y)$ in ξ, i.e. there is a $w \in \mathbb{N}^*$ such that $\rho(y) = \rho(x)w$.

(h) $(\xi, \rho) \models \varphi_2$ iff $\rho(U)$ is the set of all positions w in ξ such that $\xi|_w = \sigma(\alpha, \beta)$.

(i) $(\xi, \rho) \models \varphi_3$ iff for every node in ξ labeled by σ, none of its child nodes is labeled by γ.

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