10. Übung (11. July 2018)

Formale Baumsprachen

Task 22 (path languages [Com+08, Exercise 2.8])

Definition. Let Σ be a ranked alphabet. For every $\xi = \sigma(\xi_1, ..., \xi_k) \in \mathrm{T}_\Sigma$, the *set of paths of* ξ is recursively defined by

$$\begin{aligned} \mathrm{Paths}(\sigma) &= \{\sigma\}, && \text{and} \\ \mathrm{Paths}(\sigma(\xi_1, ..., \xi_k)) &= \{\sigma\} \cdot \bigcup_{i \in [k]} \mathrm{Paths}(\xi_k) && \text{if } k > 0. \end{aligned}$$

(a) Prove that $\mathrm{Paths}(L) = \bigcup_{\xi \in L} \mathrm{Paths}(\xi)$ is regular for every regular tree language L.

(b) What about the converse?

Task 23 (Myhill-Nerode theorem for trees (I))

Give an example for a bu-det fta $\mathcal{A}$ with $\sim_{\mathcal{A}} \subsetneq \equiv_{L(\mathcal{A})}$.

Task 24 (Myhill-Nerode theorem for trees (II))

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $L \subseteq T_\Sigma$ be the language consisting of all trees with exactly as many αs as βs. Use the Myhill-Nerode theorem to show that L is not recognizable.

References

[Com+08] Hubert Comon, Max Dauchet, Rémi Gilleron, Christof Löding, Florent Jacquemard, Denis Lugiez, Sophie Tison, and Marc Tommasi. *Tree Automata Techniques and Applications*. Nov. 18, 2008. url: `http://tata.gforge.inria.fr/`.