

## Formale Baumsprachen

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### **Task 18 (relabelings)**

(a) Show that any relabeling preserves the image under  $\text{pos}$ .

Let  $\Sigma$  and  $\Delta$  be ranked alphabets.

(b) Under which conditions is there a relabeling between trees over  $\Sigma$  and trees over  $\Delta$ ?

(c) Let  $\tau$  be a relabeling between trees over  $\Sigma$  and trees over  $\Delta$ . Now consider  $\sigma \in \Sigma$ ,  $\xi \in T_\Sigma$ , and  $L \subseteq T_\Sigma$ . Quantify  $\tau$  in the following expressions:

- (i)  $\tau(\sigma)$ ,                      (ii)  $\tau(\xi)$ , and                      (iii)  $\tau(L)$ .

### **Task 19 (construction of Bar-Hillel, Perles, and Shamir)**

Consider the ranked alphabet  $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)}\}$  and the fta  $\mathcal{A} = (Q, \Sigma, \delta, F)$  where  $Q = \{e, o\}$ ,  $F = \{e\}$ , and

$$\delta_\alpha = \delta_\beta = \delta_\gamma = \{(\varepsilon, o)\}, \quad \delta_\sigma = \{(oo, e), (ee, o)\}.$$

Moreover, let us assume an fsa  $\mathcal{B} = (P, \Delta, p, \mu, G)$  where  $\Delta = \Sigma^{(0)} \setminus \{\lambda\}$ ,  $P = \{p, r\}$ ,  $G = \{r\}$ , and

$$\mu = \{(p, \alpha, p), (p, \beta, r), (r, \beta, r)\}.$$

Using the technique from the lecture, construct an fta  $\mathcal{A}'$  such that

$$L(\mathcal{A}') = L(\mathcal{A}) \cap \text{yield}_\lambda^{-1}(L(\mathcal{B})).$$