Formale Baumsprachen

*Task 14 (*yield(Rec) *and* CF*)*

(a) Let $\Delta = \{(_a,)_a, (_b,)_b\}$. Give a context-free grammar that generates D_Δ (i.e. the Dyck language of well-bracketed strings).

(b) Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \gamma^{(0)}\}$ and G be an rtg over Σ with initial symbol Z and productions

$$Z \to \alpha + \beta + \gamma + \sigma(A, Z) + \sigma(Z, B) \quad A \to \alpha + \gamma + \sigma(A, A) \quad B \to \beta + \gamma + \sigma(B, B).$$

Describe the string languages $\mathrm{yield}_\alpha(L(G))$, $\mathrm{yield}_\beta(L(G))$, and $\mathrm{yield}_\gamma(L(G))$.

(c) Give an rtg H in normal form with $D_\Delta = \mathrm{yield}_e(L(H))$ (see Task 14 (a)) for some e.

(d) Using the construction from the lecture, transform the rtg G (see Task 14 (b)) into a context-free grammar G' such that $L(G') = \mathrm{yield}_\gamma(L(G))$. Is there a simpler context-free grammar generating the same language?

Task 15 (complement of a string automaton)

(a) Give a non-deterministic string automaton $\mathcal{M}$ whose language is not complemented by complementing its final states.

(b) Give a string automaton $\mathcal{M}'$ such that $L(\mathcal{M}') = \Sigma^* \setminus L(\mathcal{M})$.

Task 16 (closure of Rec *under intersection, union, and complement)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the following recognizable tree languages

$$L_1 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathrm{pos}(\xi)\colon w \in \{2\}^* \text{ if and only if } \xi(w) \in \{\sigma, \alpha\}\} \text{ and}$$
$$L_2 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathrm{pos}(\xi)\colon \xi(w) = \alpha \text{ only if } |w| \equiv 0 \pmod 2\}.$$

Find finite representations for the following languages:

(a) L_1 (b) L_2 (c) $L_1 \cup L_2$ (d) $L_1 \cap L_2$ (e) $T_\Sigma \setminus L_1$

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