Formale Baumsprachen

Task 12 (relatedness)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{Z, A, B, C\}$ and

$$P = \{\ Z \to \sigma(\sigma(A,B),C), \quad Z \to B, \quad A \to \alpha, \quad A \to B,$$
$$B \to \beta, \quad B \to A, \quad B \to C, \quad C \to C\ \}\ .$$

(a) Use the construction from the lecture to give a regular tree grammar in normal form equivalent to G.

(b) Give a bu-det fta that is related to the normal form regular tree grammar constructed in Exercise 12 (a).

(c) Give a regular tree grammar that is related to the normal form bu-det fta $\mathcal{M} = (Q, \Sigma, \tau, \{q_0\})$ where $Q = \{0, 1\}$, $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$, $q_0 = 0$, and τ is given by

$$\tau_\alpha() = 1, \qquad \tau_\beta() = 0, \text{ and} \qquad \tau_\sigma(p,q) = (p+q)\,\%\,2 \qquad \text{for each } p, q \in Q.$$

Task 13 (tree manipulation)

Let Σ be a ranked alphabet and H be a set. Prove or refute the following statements for every $\xi \in \mathrm{T}_\Sigma(H)$:

(a) $\forall w \in \mathrm{pos}(\xi)\colon \mathrm{pos}(\xi|_w) \subseteq \mathrm{pos}(\xi)$,

(b) $\forall w \in \mathrm{pos}(\xi)\colon \mathrm{sub}(\xi|_w) \subseteq \mathrm{sub}(\xi)$,

(c) $\forall w \in \mathrm{pos}(\xi), \zeta \in \mathrm{T}_\Sigma(H)\colon \mathrm{size}(\xi[\zeta]_w) = \mathrm{size}(\xi) + \mathrm{size}(\zeta) - \mathrm{size}(\xi|_w)$.

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