3. Übung (May 2, 2018)

Formale Baumsprachen

Task 6 (bu-det fta)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be ranked alphabets. Give deterministic bu-ta $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ that recognize L_1, L_2, and L_3, respectively, where

(a) $L_1 = \{\xi \in T_\Sigma \mid \xi$ contains at least one α and one $\beta\}$,

(b) $L_2 = \{\xi \in T_\Sigma \mid \xi$ contains an even number of α symbols$\}$, and

(c) $L_3 = \{\sigma(t_1, \sigma(t_2, ...\sigma(t_n, \alpha)...)) \in T_\Delta \mid n \in \mathbb{N}, t_1, ..., t_n \in T_{\{\gamma^{(1)}, \alpha^{(0)}\}}\}$.

Task 7 (string automata)

Recall the concept of string automata. Let Σ be an alphabet and $\# \notin \Sigma$. We define the ranked alphabet $\Sigma_\# = \Sigma_\#^{(0)} \cup \Sigma_\#^{(1)}$ where $\Sigma_\#^{(0)} = \{\#\}$ and $\Sigma_\#^{(1)} = \Sigma$. Moreover, we define the $\Sigma_\#$-algebra (Σ^*, θ) where $\theta(\#) = \varepsilon$ and $\theta(a)(w) = wa$ for every $a \in \Sigma$ and $w \in \Sigma^*$.

(a) Show that Σ^* is initial in the class of $\Sigma_\#$-algebras.

(b) We consider $\Sigma = \{a, b\}$ and the language $L = \{a^n b^m \mid n, m \in N\}$. Sketch the diagram of a total deterministic finite-state automaton accepting L and model the transition table using a finite $\Sigma_\#$-algebra Q. How can we interpret the uniquely determined homomorphism $h\colon \Sigma^* \to Q$?

(c) Convince yourself that any total deterministic finite-state automaton can be modeled as a quadruple $\mathcal{A} = (Q, \Sigma, \theta, F)$ where (Q, θ) is a finite $\Sigma_\#$-algebra and $F \subseteq Q$. Define the language accepted by $\mathcal{A}$ using the homomorphism $h\colon \Sigma^* \to Q$.

Task 8 (universal algebra)

(a) Show that the mapping sub (restricted to T_Σ) is a homomorphism. Start by giving the target algebra.

(b) Show that the principle of proof by structural induction is correct by applying the above concepts from universal algebra.

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