2nd tutorial (April 25, 2018)

Formale Baumsprachen

Task 3 (proof by structural induction)

Let Σ be a ranked alphabet and H be a set. Prove or refute the following statements for every $\xi, \zeta \in \mathrm{T}_\Sigma(H)$, and $w \in \mathrm{pos}(\xi)$:

(a) $\xi(w) = \xi|_w(\varepsilon)$,

(b) $(\xi[\zeta]_w)|_w = \zeta$.

Task 4 (proof of the size-height-lemma)

Prove the following lemma.

Lemma. *Let Σ be a ranked alphabet. Then there is an integer $c \geq 1$ such that, for every $\xi \in \mathrm{T}_\Sigma$,* $\mathrm{size}(\xi) \leq c^{\mathrm{height}(\xi)}$.

Task 5 (universal algebra)

(a) Recall the following concepts: Σ-algebra, Σ-homomorphism, initial Σ-algebra in a class $\mathcal{K}$, and Σ-term algebra.

(b) Show that the mappings height, size, and sub (restricted to T_Σ) are homomorphisms. Start by giving the target algebra for each of them. What is the problem concerning sub?

(c) Show that the principle of proof by structural induction is correct, applying the above concepts from universal algebra.

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