Formale Baumsprachen

Task 1 (definition by structural induction)

Let Σ be a ranked alphabet, $\xi, \xi_1, \ldots, \xi_k \in \mathrm{T}_\Sigma$, and $\zeta \in \mathrm{T}_\Sigma(X_k)$. Define the following functions by structural induction:

(a) $\mathrm{yield}(\xi)$, the sequence of leaves in ξ from left to right; and

(b) $\zeta[\xi_1, \ldots, \xi_k]$, the tree obtained from ζ by replacing every occurrence of x_i by ξ_i for every $i \in \{1, \ldots, k\}$.

In the lecture we defined trees as well-formed expressions. An alternative definition characterises a tree as a tuple (t, φ) where, intuitively, t is a set of *Gorn addresses* that is closed under certain operations and φ assigns a symbol from some alphabet Δ to every element of t.

(c) Give a formal definition of trees over Δ in the above sense.

Formally define the following characteristics of trees in the sense of Task 1 (c):

(d) height,

(e) size,

(f) set of positions,

(g) subtree at a position,

(h) set of subtrees, and

(i) label at a position.

Task 2 (proof by structural induction)

Let Σ be a ranked alphabet and H be a set. Prove or refute the following statements for every $\xi \in \mathrm{T}_\Sigma(H)$:

(a) $\mathrm{height}(\xi) = 1 + \max\{|w| \mid w \in \mathrm{pos}(\xi)\}$, and

(b) $|\mathrm{pos}(\xi)| = |\mathrm{sub}(\xi)|$.

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