
Formale Übersetzungsmodelle

**Task 18 (decomposition of TOP)**

Let \( \Sigma = \{ \gamma^{(1)}, \alpha^{(0)} \} \) and \( \Delta = \{ \sigma^{(2)}, O^{(1)}, E^{(1)}, \alpha^{(0)} \} \) be ranked alphabets and \( \xi = \gamma(\gamma(\alpha)) \in T_{\Sigma} \).

(a) Give a td-tt \( T \) such that \( \tau(T) \) transforms every tree in \( T_{\Sigma} \) into a tree in \( T_{\Delta} \) such that each \( \gamma \) is replaced by \( \sigma \) where the subtree of \( \gamma \) is copied and, starting with \( O \) at the top, alternately \( O \) and \( E \) are inserted before each symbol.

Give a derivation of \( T \) for \( \xi \).

(b) Give a top-down tree homomorphism \( H \) and a linear top-down tree transducer \( T' \) such that \( \tau(T) = \tau(H) \circ \tau(T') \).

Give derivations of \( H \) and \( T' \) for \( \xi \).

**Task 19 (generalized sequential machines and top-down tree transducers)**

GSM is the class of string transformations \( \tau \subseteq \Sigma^{*} \times \Delta^{*} \) that are be induced by some gsm.

(a) Give formal definitions for the syntax and derivation relation of a gsm, and the string transformation induced by a gsm.

(b) Prove by construction that GSM is closed under composition.

**Hint:** Use a product construction where the right hand side of a rule of the first gsm is processed by the second gsm (pipelining).

Let \( G = (Q, \Sigma, \Delta, q_0, F, R) \) be a gsm.

(c) Give a gsm \( G^R \) such that \( \tau(G^R) = \{(w^R_l, w^R_r) \mid (w_l, w_r) \in \tau(G)\} \) where \( w^R \) denotes the reverse of \( w \).

(d) Give a td-tt that simulates the run of \( G \) on the nodes of monadic trees from root to front.