
Formale Übersetzungsmodelle

Task 15 (BOT ; HOM \subseteq BOT)

Let $B = (Q, \Sigma, \Delta, F, R)$ be a bu-tt and $H = (\{*\}, \Delta, \Omega, \{*\}, R_H)$ a tree homomorphism. Also let $X_{\max} = \{x_i \mid i \in [\max \text{rank}(\Sigma)]\}$. Define the bottom-up tree homomorphism $H' = (\{*\}, \Delta, \Omega \cup X_{\max}, \{*\}, R_H)$ and the bottom-up tree transducer $\hat{B} = (Q, \Sigma, \Omega, F, \hat{R})$ where

$$\begin{aligned} \sigma(q_1(x_1), \dots, q_k(x_k)) &\rightarrow q(u') \in R \wedge u'[* (x_1), \dots, *(x_k)] \Rightarrow_{H'}^* *(t') \\ \Leftrightarrow \sigma(q_1(x_1), \dots, q_k(x_k)) &\rightarrow q(t') \in \hat{R}. \end{aligned}$$

Show that for every $s \in T_\Sigma$, $q \in Q$, and $t \in T_\Delta$ the following equivalence holds:

$$s \Rightarrow_{\hat{B}}^* q(t) \Leftrightarrow \exists u \in T_\Delta : s \Rightarrow_B^* q(u) \wedge u \Rightarrow_H^* *(t).$$

Task 16 (regular tree grammars)

Consider the ranked alphabet $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$, the tree $\xi = \sigma(\sigma(\alpha, \alpha), \sigma(\alpha, \alpha)) \in T_\Sigma$, and the regular tree grammar $G = (\{S, A\}, \Sigma, S, R)$ where

$$R: \quad S \rightarrow A \qquad S \rightarrow \sigma(S, S) \qquad A \rightarrow \sigma(\alpha, S) \qquad A \rightarrow \alpha.$$

- Give a derivation and the corresponding derivation tree of ξ in G .
How many derivation trees of ξ do exist in G ?
- Give an RTG H and a tree ζ such that ζ has infinitely many derivations in H .
- Give an RTG G' such that G' is in normal form and $L(G') = L(G)$.
Give a derivation tree of ξ in G' .
- Prove by construction that for every RTG G there exists an RTG G' in normal form such that $L(G') = L(G)$.