
Formale Übersetzungsmodelle

**Task 15 (BOT; HOM \subseteq BOT)**

Let $B = (Q, \Sigma, \Delta, F, R)$ be a bu-tt and $H = (\{\ast\}, \Delta, \Omega, \{\ast\}, R_H)$ a tree homomorphism. Also let $X_{max} = \{x_i \mid i \in \text{max rank}(\Sigma)\}$. Define the bottom-up tree homomorphism $H' = (\{\ast\}, \Delta, \Omega \cup X_{max}, \{\ast\}, R_H)$ and the bottom-up tree transducer $\hat{B} = (Q, \Sigma, \Omega, F, \hat{R})$ where

\[
\sigma(q_1(x_1), \ldots, q_k(x_k)) \rightarrow q(u') \land u'[\ast(x_1), \ldots, \ast(x_k)] \Rightarrow_H \ast(t')
\]

\[
\Rightarrow \sigma(q_1(x_1), \ldots, q_k(x_k)) \rightarrow q(t') \in \hat{R}.
\]

Show that for every $s \in T_\Sigma$, $q \in Q$, and $t \in T_\Delta$ the following equivalence holds:

\[
s \Rightarrow_B^* q(t) \iff \exists u \in T_\Delta : s \Rightarrow_B^* q(u) \land u \Rightarrow_H^* \ast(t).
\]

**Task 16 (regular tree grammars)**

Consider the ranked alphabet $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$, the tree $\xi = \sigma(\sigma(\alpha, \alpha), \sigma(\alpha, \alpha)) \in T_\Sigma$, and the regular tree grammar $G = (\{S, A\}, \Sigma, S, R)$ where

\[
R : S \rightarrow A \quad S \rightarrow \sigma(S, S) \quad A \rightarrow \sigma(\alpha, S) \quad A \rightarrow \alpha.
\]

(a) Give a derivation and the corresponding derivation tree of $\xi$ in $G$.

(b) How many derivation trees of $\xi$ do exist in $G$?

(c) Give an RTG $H$ and a tree $\zeta$ such that $\zeta$ has infinitely many derivations in $H$.

(d) Give an RTG $G'$ such that $G'$ is in normal form and $L(G') = L(G)$.

Give a derivation tree of $\xi$ in $G'$.

(d) Prove by construction that for every RTG $G$ there exists an RTG $G'$ in normal form such that $L(G') = L(G)$. 

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