

## Formale Übersetzungsmodelle

### Task 13 (decompositions of BOT)

Let  $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ . Consider the bu-tt  $M = (\{q_0, q_1, p\}, \Sigma, \Sigma, \{q_0\}, R)$ , where

$$R: \begin{array}{lll} \alpha \rightarrow q_0(\alpha), & \gamma(p(x_1)) \rightarrow p(\gamma(x_1)), & \sigma(q_0(x_1), p(x_2)) \rightarrow q_1(\sigma(x_2, x_1)), \\ \alpha \rightarrow p(\alpha), & & \sigma(q_1(x_1), p(x_2)) \rightarrow q_0(\sigma(x_1, x_2)). \end{array}$$

- (a) Describe  $\tau(M)$ .
- (b) Construct, according to the decomposition result from the lecture, a relabeling bu-tt  $M_1$ , an fta  $M_2$ , and a homomorphism bu-tt  $M_3$  such that  $\tau(M) = \tau(M_1); \tau(M_2); \tau(M_3)$ .
- (c) Illustrate the transformation  $\tau(M_1); \tau(M_2); \tau(M_3)$  with the input tree  $\sigma(\alpha, \gamma\alpha) \in T_\Sigma$ .
- (d) Elaborate on the construction of the qrel bu-tt in the proof of  $\text{BOT} \subseteq \text{QREL}; \text{HOM}$ .
- (e) Give a qrel bu-tt  $M'_1$  and a homomorphism bu-tt  $M'_2$  such that  $\tau(M) = \tau(M'_1); \tau(M'_2)$ .

### Task 14 (bimorphism characterization of BOT)

Recall the decomposition result  $\text{BOT} \subseteq \text{REL}; \text{FTA}; \text{HOM}$ . For every bottom-up tree transducer  $B$  we can construct a bottom-up tree relabeling  $B_1$ , a bottom-up finite state tree automaton  $B_2$ , and a bottom-up tree homomorphism  $B_3$  such that  $\tau(B) = \tau(B_1); \tau(B_2); \tau(B_3)$ .

- (a) Show that  $(\tau(B_1))^{-1} \in \text{HOM}$ .
- (b) Give a bimorphism characterization of BOT, i.e., give a formal definition of (the syntax of) bimorphisms, and its induced tree transformation. Show that the class of tree transformations induced by bimorphisms subsumes BOT.
- (c) Consider the bu-tt  $B = (\{*, q, q_f\}, \Sigma, \Sigma, \{q_f\}, R)$  where  $\Sigma = \{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \sigma^{(2)}\}$  and

$$R: \begin{array}{llll} \alpha \rightarrow q(\alpha) & \beta \rightarrow *(\beta) & \gamma(*(\alpha)) \rightarrow *(\gamma(\alpha)) & \sigma(*(\alpha), q(\alpha)) \rightarrow q_f(\alpha) \\ \alpha \rightarrow *(\alpha) & & \gamma(q(\alpha)) \rightarrow q(\gamma(\alpha)) & \sigma(*(\alpha), *(\alpha)) \rightarrow *(\sigma(\alpha, \alpha)) \end{array}$$

Give a bimorphism  $\mathcal{B} = (A, \varphi, \psi)$  such that  $\tau(B) = \tau(\mathcal{B})$ .

Give a derivation of  $\xi = \sigma(\gamma(\beta), \gamma(\alpha))$  in  $B$ .

Give a tree  $\zeta \in T_\Omega$  such that  $\zeta \in L(A)$  and  $\varphi(\zeta) = \xi$ .