Formale Übersetzungsmodelle

Task 10 (nondeterminism and determinism)

Let $\Sigma = \{\top^{(0)}, \bot^{(0)}, \neg^{(1)}, \wedge^{(2)}\}$ be a ranked alphabet. Note that the trees over Σ represent a subset of the formulae of propositional logic.

(a) Give a bu-tt M that eliminates double negations, i.e., for every $\xi \in T_\Sigma$, M reduces every subtree of the form $\neg(\neg(\xi))$ to ξ.

(b) Give a deterministic bu-tt M_{det}, such that $\tau(M_{\mathrm{det}}) = \tau(M)$.

(c) For any given bu-tt M', is there a deterministic bu-tt M'_{det} such that $\tau(M'_{\mathrm{det}}) = \tau(M')$?

(d) What restriction is required from M' in order for M'_{det} to exist?

Task 11 (powerset construction)

Let $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$ be a ranked alphabet. Consider the bottom-up finite state tree automaton $N = (\{q_0, q_1\}, \Sigma, \Sigma, \{q_0\}, R)$ where R is given by

$$\alpha \to q_0(\alpha) \qquad \text{and} \qquad \sigma(q_i(x_1), q_j(x_2)) \to q_{1-k}(\sigma(x_1, x_2))$$
for every $i, j \in \{0, 1\}$ and $k \in \{i, j\}$.

(a) Determine the tree language of N.

(b) Use the powerset construction to give a deterministic bottom-up finite state tree automaton N_{det} such that $\tau(N) = \tau(N_{\mathrm{det}})$.

Task 12 (bounded growth property)

Prove the following statement [cf. Eng75, Lem. 1.1, p. 205]:

Claim. There exists a $c \in \mathbb{N}$ such that for every $(s, t) \in \tau(M)$ holds $\mathsf{height}(t) \le c \cdot \mathsf{height}(s)$.

References

[Eng75] J. Engelfriet. “Bottom-up and top-down tree transformations—a comparison”. In: *Mathematical systems theory* 9.2 (1975), pp. 198–231. issn: 0025-5661. doi: `10.1007/BF01704020`.

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