2. Übung (19. Oktober 2017)

Formale Übersetzungsmodelle

**Task 3 (relabeling and checking)**

(a) Give a bu-\(\tau\) \(M_1\) that, for every tree \(\xi \in T_\Sigma\), enhances for every position \(w \in \text{pos}(\xi)\) the label at \(w\) with the last digit of \(w\).

(b) Let \(\gamma \in \Sigma\). Give a bu-\(\tau\) \(M_2\) that, for every tree \(\xi \in T_\Sigma\), replaces the first occurrence (according to depth-first order) of \(\gamma\) in \(\xi\) by \(\gamma_f\) without changing the rest of \(\xi\).

\[
\begin{array}{ccc}
\sigma & \sigma, \varepsilon \\
\gamma & \beta & (\gamma, 1) (\beta, 2) \\
\alpha & (\alpha, 1) \\
\end{array}
\]

(a) transformation \(\tau(M_1)\)

\[
\begin{array}{ccc}
\sigma & \sigma \\
\gamma & \gamma & \gamma_f \\
\alpha & \beta & \alpha & \beta \\
\end{array}
\]

(b) transformation \(\tau(M_2)\)

**Task 4 (proof by structural induction)**

Let \(A\) be a set, \(\Sigma\) be a ranked alphabet, \(\xi, \zeta \in T_\Sigma(A)\), and \(w \in \text{pos}(\xi)\). Prove or refute the following statements:

(a) \(\xi(w) = \xi|_w(\varepsilon)\).

(b) \((\xi[\zeta]|_w)|_w = \zeta\).

(c) \(|\text{pos}(\xi)| = |\text{sub}(\xi)|\).

(d) \(\text{height}(\xi) = 1 + \max\{|\rho| \mid \rho \in \text{pos}(\xi)\}\).

**Task 5 (generalized sequential machines and bu-\(\tau\))**

Let \(G = (Q, \Sigma, \Delta, q_0, F, R)\) be a gsm. Give bu-tts that simulate the run of \(G\)

(a) on the nodes of monadic trees from front to root.

(b) on the front of trees from left to right.

**Note:** The tutorial’s time might not suffice to present all solutions. Please prepare to ask for the solutions you are most interested in.