Task 5 (generalized sequential machines and bu-tt)

Let $G = (Q, \Sigma, \Delta, q_0, F, R)$ be a gsm. Give bu-tts that simulate the run of $G$

(a) on the nodes of monadic trees from front to root.

(b) on the front of trees from left to right.

Solution for Task 5

(a) Trees have the form $\sigma_1(\sigma_2(...(\sigma_k(#))...))$ where $\sigma_1, ..., \sigma_k \in \Sigma$. We define

$$M_a = (Q, \Sigma', \Delta', F, R_a)$$

where $\Sigma' = \{\sigma^{(1)} \mid \sigma \in \Sigma\} \cup \{#^{(0)}\}$, $\Delta' = \{\delta^{(1)} \mid \delta \in \Delta\} \cup \{#^{(0)}\}$, and

$$R_a = \{\sigma(p(x_1)) \rightarrow q(\delta_1(...\delta_k(x_1)...)) \mid (p\sigma \rightarrow \delta_1...\delta_kq) \in R\} \cup \{# \rightarrow q_0(#)\}$$

(b) We extend the usual construction by Bar-Hillel, Perles, and Shamir [1, Theorem 8.1]: We are given some ranked alphabet $\Sigma'$ and $\Delta'$ whose symbols of rank 0 are $\Sigma$ and $\Delta$, respectively, and symbols of any other rank are the same. Since $G$ can replace one input symbol by several output symbols (or none), we introduce a new output symbol $@_l$ for every output length $l$ of the rules in $G$. Note that $@_0$ needs to be deleted by the “front” function.

The constructed bu-tt has to guess the previous state at the leaves, since it can only see one input symbol (the leaf) there. At each symbol we annotate a tuple of states from $G$ as the state of our bu-tt, the left projection represents the source state and the right projection represents the target state in $G$. At each inner node, the neighbouring states of the children are then checked to be equal (right component of $i$-th child equals left component of $(i + 1)$-th child).

We define

$$M_b = (Q \times Q, \Sigma', \Delta' \cup \{@_l \mid l \in \text{max}\{|w| \mid (p\alpha \rightarrow wq) \in R\}\}, \{(q_0, q_f) \mid q_f \in F\}, R_b)$$

where

$$R_b = \{\alpha \rightarrow (p, q)(@_l(\delta_1, ..., \delta_k)) \mid p\alpha \rightarrow \delta_1...\delta_kq \in R\}$$

$$\cup \{\sigma((q_0, q_1)(x_1), (q_1, q_2)(x_2), ..., (q_{k-1}, q_k)(x_k)) \rightarrow (q_0, q_k)(\sigma(x_1, ..., x_k)) \mid k \in \mathbb{N}, \sigma \in \Sigma'^{(k)}, q_0, ..., q_k \in Q\}$$

References