Task 5 (generalized sequential machines and bu-tt)

Let $G = (Q, \Sigma, \Delta, q_0, F, R)$ be a gsm. Give bu-tts that simulate the run of G

(a) on the nodes of monadic trees from front to root.

(b) on the front of trees from left to right.

Solution for Task 5

(a) Trees have the form $\sigma_1(\sigma_2(...(\sigma_k(\#))...))$ where $\sigma_1, ..., \sigma_k \in \Sigma$. We define

$$M_a = (Q, \Sigma', \Delta', F, R_a)$$

where $\Sigma' = \{\sigma^{(1)} \mid \sigma \in \Sigma\} \cup \{\#^{(0)}\}$, $\Delta' = \{\delta^{(1)} \mid \delta \in \Delta\} \cup \{\#^{(0)}\}$, and

$$R_a = \{\sigma(p(x_1)) \to q(\delta_1(...\delta_k(x_1)...)) \mid (p\sigma \to \delta_1 \cdots \delta_k q) \in R\} \cup \{\# \to q_0(\#)\}$$

(b) We extend the usual construction by Bar-Hillel, Perles, and Shamir [1, Theorem 8.1]: We are given some ranked alphabet Σ' and Δ' whose symbols of rank 0 are Σ and Δ, respectively, and symbols of any other rank are the same. Since G can replace one input symbol by several output symbols (or none), we introduce a new output symbol $@_l$ for every output length l of the rules in G. Note that $@_0$ needs to be deleted by the "front" function.

The constructed bu-tt has to guess the previous state at the leaves, since it can only see one input symbol (the leaf) there. At each symbol we annotate a tuple of states from G as the state of our bu-tt, the left projection represents the source state and the right projection represents the target state in G. At each inner node, the neighbouring states of the children are then checked to be equal (right component of i-th child equals left component of $(i+1)$-th child).

We define

$$M_b = (Q \times Q, \Sigma', \Delta' \cup \{@_l \mid l \in [\max\{|w| \mid (p\alpha \to wq) \in R\}]\}, \{(q_0, q_f) \mid q_f \in F\}, R_b)$$

where

$$\begin{aligned} R_b = \{&\alpha \to (p,q)(@_l(\delta_1, ..., \delta_\ell)) \mid p\alpha \to \delta_1 \cdots \delta_\ell q \in R\} \\ \cup \{&\sigma((q_0,q_1)(x_1),(q_1,q_2)(x_2),...,(q_{k-1},q_k)(x_k)) \to (q_0,q_k)(\sigma(x_1,...,x_k)) \\ &\mid k \in \mathbb{N}, \sigma \in \Sigma'^{(k)}, q_0, ..., q_k \in Q\} \end{aligned}$$

References

[1] Yehoshua Bar-Hillel, M. Perles, and Eli Shamir. "On Formal Properties of Simple Phrase Structure Grammars". In: *Zeitschrift für Phonetik, Sprachwissenschaft und Kommunikationsforschung* 14 (Jan. 1, 1961). Reprinted in Y. Bar-Hillel. (1964). *Language and Information: Selected Essays on their Theory and Application*, Addison-Wesley 1964, 116–150, pp. 143–172.