Formale Baumsprachen

*Task 24 (*Rec = *MSO-definable)*

(a) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Consider the bottom-up deterministic fta $\mathcal{A} = (Q, \Sigma, \delta, F)$ with $Q = \{0, 1\}$, $F = \{1\}$, and $\delta_\sigma(q_1, q_2) = \max\{q_1, q_2\}$ for every $q_1, q_2 \in Q$, $\delta_\gamma(q) = 1$ for every $q \in Q$, and $\delta_\alpha() = 0$. Use the construction from the lecture to show that $L(\mathcal{A})$ is MSO-definable.

(b) Recall the following Lemma from the lecture:

Lemma. Let Σ be a ranked alphabet and $\mathcal{V} \subseteq_{\text{fin}} \mathcal{V}_1$. Then $T^{\text{v}}_{\Sigma_\mathcal{V}}$ is recognizable.

In the proof we required a family of languages $(L_x \mid x \in \mathcal{V})$ where for every $x \in \mathcal{V}$:

$$L_x = \{\xi \in T_{\Sigma_\mathcal{V}} \mid x \text{ occurs exactly once in } \xi\}.$$

Construct an automaton $\mathcal{A}_x$ for every $x \in \mathcal{V}$ such that $L(\mathcal{A}_x) = L_x$.

(c) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\varphi = \exists x.\text{label}_\gamma(x)$. Use the construction from the lecture to show that $L(\varphi)$ is recognizable.

Task 25 (consultation)

You may ask the tutorial instructor questions about the contents of the entire lecture.

`http://www.orchid.inf.tu-dresden.de` `Tobias.Denkinger@tu-dresden.de`