

Formale Baumsprachen

Task 23 (monadic second-order logic on trees)

- (a) Define macros for conjunction, implication, first and second order universal quantification, an edge relation, a leaf predicate, and a root predicate.

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the MSO-formula

$$\varphi = \exists U. \neg \exists x. \exists y. \text{edge}_2(x, y) \wedge \text{label}_\sigma(y) \wedge x \in U$$

over Σ where $x, y \in \mathcal{V}_1$ and $U \in \mathcal{V}_2$.

- (b) Calculate $\text{Free}(\varphi)$ and $\text{Bound}(\varphi)$ using the definitions from the lecture.
 (c) Is φ closed?

Consider the tree $\xi = \sigma(\gamma(\alpha), \beta)$ and the following functions:

$$\begin{aligned} \rho_1: x \mapsto \varepsilon, x' \mapsto 1, y \mapsto 11, y' \mapsto 2, \\ \rho_2: x \mapsto \varepsilon, x' \mapsto \varepsilon, \bar{x} \mapsto 1, y \mapsto 11, y' \mapsto 2, \\ \rho_3: X \mapsto \{\varepsilon, 1\}, Y \mapsto \{11, 2\}, \text{ and} \\ \rho_4: X \mapsto \emptyset, Y \mapsto \{1, 2, 3\}, x \mapsto \varepsilon. \end{aligned}$$

- (d) Which of the functions ρ_1, \dots, ρ_4 are assignments for ξ ? Give the appropriate sets of variables.
 (e) Encode the assignments from Task 23 (d) as trees.
 (f) Which of the trees obtained in Task 23 (e) are valid?

Let $\mathcal{V} = \{x, y, U\}$. Construct MSO-formulas φ_1, φ_2 , and φ_3 such that $\text{Free}(\varphi_1) \cup \text{Free}(\varphi_2) \cup \text{Free}(\varphi_3) \subseteq \mathcal{V}$ and for every $\xi \in T_\Sigma$ and \mathcal{V} -assignment ρ for ξ :

- (g) $(\xi, \rho) \models \varphi_1$ iff there is a downward path from the node $\rho(x)$ to the node $\rho(y)$ in ξ , i.e. there is a $w \in \mathbb{N}^*$ such that $\rho(y) = \rho(x)w$.
 (h) $(\xi, \rho) \models \varphi_2$ iff $\rho(U)$ is the set of all positions w in ξ such that $\xi|_w = \sigma(\alpha, \beta)$.
 (i) $(\xi, \rho) \models \varphi_3$ iff for every node in ξ labeled by σ , none of its child nodes is labeled by γ .