# Formale Baumsprachen

### *Task 18 (local tree languages [Com+08, Exercise 2.5])*

**Definition.** Let $\Sigma$ be a ranked alphabet. For every $\xi \in T_\Sigma$, the *fork of* $\xi$ is the set

$$\mathsf{fork}(\xi) = \{(\sigma, \sigma_1 \cdots \sigma_k) \in \Sigma \times \Sigma^* \mid \rho \in \mathsf{pos}(\xi), \xi(\rho) = \sigma, k = \mathsf{rank}(\sigma), \forall i \in [k] \colon \xi(\rho i) = \sigma_i\}.$$

A tree language $L \subseteq T_\Sigma$ is called *local* if there are sets $F \subseteq \Sigma$ and $G \subseteq \mathsf{fork}(T_\Sigma)$ such that $\xi \in L$ iff $(\xi(\varepsilon) \in F) \wedge (\mathsf{fork}(\xi) \subseteq G)$. □

(a) Proof that every local tree language is regular.

(b) Proof that a language is local iff it is the set of parse trees of a context-free string grammar.

(c) Proof that every regular tree language is the image of a local tree language by an alphabetic homomorphism.

### *Task 19 (path languages [Com+08, Exercise 2.8])*

**Definition.** Let $\Sigma$ be a ranked alphabet. For every $\xi = \sigma(\xi_1, \ldots, \xi_k) \in T_\Sigma$, the *set of paths of* $\xi$ is recursively defined by

$$\mathsf{Paths}(\xi) = \sigma(\mathsf{Paths}(\xi_1), \ldots, \mathsf{Paths}(\xi_k)).$$

(a) Proof that $\mathsf{Paths}(L) = \bigcup_{\xi \in L} \mathsf{Paths}(\xi)$ is regular for every regular tree language $L$.

(b) What about the converse?

### *References*

[Com+08] Hubert Comon, Max Dauchet, Rémi Gilleron, Christof Löding, Florent Jacquemard, Denis Lugiez, Sophie Tison, and Marc Tommasi. *Tree Automata Techniques and Applications*. Nov. 18, 2008. url: `http://tata.gforge.inria.fr/`.