

Formale Baumsprachen

Task 12 (complement of a string automaton)

- (a) Give a non-deterministic string automaton \mathcal{M} whose language is not complemented by complementing its final states.
- (b) Give a string automaton \mathcal{M}' such that $L(\mathcal{M}') = \Sigma^* \setminus L(\mathcal{M})$.

Task 13 (closure of Rec under intersection, union, and complement)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the following recognizable tree languages

$$L_1 = \{\xi \in T_\Sigma \mid \text{for every } w \in \text{pos}(\xi): w \in \{2\}^* \text{ if and only if } \xi(w) \in \{\sigma, \alpha\}\}$$

$$L_2 = \{\xi \in T_\Sigma \mid \text{for every } w \in \text{pos}(\xi): \xi(w) = \alpha \text{ only if } |w| \equiv 0 \pmod{2}\}.$$

Find finite representations for the following languages:

- (a) L_1
- (b) L_2
- (c) $L_1 \cup L_2$
- (d) $L_1 \cap L_2$
- (e) $T_\Sigma \setminus L_1$

Task 14 (concatenation and Kleene star for recognizable tree languages)

Let Σ be a ranked alphabet.

- (a) Show that $\text{Rec}(\Sigma)$ is closed under top concatenation without using the fact that it is closed under tree concatenation.
- (b) Why can we not use the closure of $\text{Rec}(\Sigma)$ under tree concatenation to prove the closure under Kleene star?

Prove or refute the following two statements:

- (c) For every $\alpha \in \Sigma^{(0)}$, the binary operation \cdot_α is associative.
- (d) $(L_1 \cdot_\alpha L_2) \cdot_\beta L_3 = L_1 \cdot_\alpha (L_2 \cdot_\beta L_3)$ for arbitrary $L_1, L_2, L_3 \in \text{Rec}(\Sigma)$ and $\alpha, \beta \in \Sigma^{(0)}$.

Let $\Delta = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet.

- (e) Using the construction from the lecture, show that $\{\sigma(\alpha, \beta)\}_\beta^* \cdot_\beta \{\alpha\} \in \text{Rec}(\Sigma)$.