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# Formale Baumsprachen

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**Task 10 (yield(Rec) and CF)**

- (a) Let  $\Delta = \{(a, )_a, (b, )_b\}$ . Give a context-free grammar that generates  $D_\Delta$  (i.e. the Dyck language of well-bracketed strings).
- (b) Let  $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \gamma^{(0)}\}$  and  $G$  be an rtg over  $\Sigma$  with initial symbol  $Z$  and productions

$$Z \rightarrow \alpha + \beta + \gamma + \sigma(A, Z) + \sigma(Z, B) \quad A \rightarrow \alpha + \gamma + \sigma(A, A) \quad B \rightarrow \beta + \gamma + \sigma(B, B).$$

Describe the string languages  $\text{yield}_\alpha(L(G))$ ,  $\text{yield}_\beta(L(G))$ , and  $\text{yield}_\gamma(L(G))$ .

- (c) Give an rtg  $H$  in normal form with  $D_\Delta = \text{yield}_e(L(H))$  (see Task 10 (a)) for some  $e$ .
- (d) Using the construction from the lecture, transform the rtg  $G$  (see Task 10 (b)) into a context-free grammar  $G'$  such that  $L(G') = \text{yield}_\gamma(L(G))$ . Is there a simpler context-free grammar generating the same language?

**Task 11 (unranked tree automata and Haskell)**

In the lecture we considered automata for ranked trees, i.e. trees where each terminal symbol has a fixed number of successors (the rank). In practice it is sometimes desirable for trees to allow an arbitrary number of successors for the terminal symbols, such trees are called unranked trees. In HTML, for example, it is possible for the `<ul>`-tag (unordered list) to have arbitrarily many successors of the form `<li>...</li>` (list item). Automata over unranked trees are called unranked tree automata.

- (a) Give a definition of unranked tree automata and deterministic unranked tree automata.
- (b) Devise a polymorphic data type `DUTA q t` in Haskell for deterministic unranked tree automata. The type variables `q` and `t` represent the states and terminal symbols, respectively.
- (c) Implement a function `recognize :: DUTA q t -> Tree t -> Bool` that returns **True** if and only if the first parameter recognizes the second parameter.
- (d) Test `recognize`.
- (e) Devise a polymorphic data type `NUTA q t` in Haskell for unranked tree automata.
- (f) Implement a function `recognize' :: NUTA q t -> Tree t -> Bool` that returns **True** if and only if the first parameter recognizes the second parameter.
- (g) Test `recognize'`.

**Note** The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.