Formale Baumsprachen

Task 7 (regular tree grammars)

(a) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Give regular tree grammars G_1 and G_2 with

- $L(G_1) = \{\xi \in T_\Sigma \mid \xi \text{ contains exactly one } \sigma\}$ and
- $L(G_2) = \{\xi \in T_\Sigma \mid \xi \text{ contains the pattern } \sigma(\_, \gamma(\_)) \text{ at least twice}\}$.

(b) Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{Z, A, B, C\}$ and

$$P = \{\ Z \to \sigma(\sigma(A, B), C), \quad Z \to B, \quad A \to \alpha, \quad A \to B, \quad B \to \beta, \quad B \to A, \quad B \to C, \quad C \to C\ \}\ .$$

Use the construction from the lecture to give a regular tree grammar in normal form equivalent to G.

Task 8 (relatedness)

(a) Give a bu-det fta that is related to the normal form regular tree grammar constructed in Exercise 7 (b).

(b) Give a regular tree grammar that is related to the normal form bu-det fta $\mathcal{M} = (Q, \Sigma, \tau, \{q_0\})$ where $Q = \{0, 1\}$, $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$, $q_0 = 0$, and τ is given by

$$\tau_\alpha() = 1, \qquad \tau_\beta() = 0, \text{ and} \qquad \tau_\sigma(p, q) = (p + q) \,\%\, 2 \qquad \text{for each } p, q \in Q.$$

Task 9 (tree manipulation)

Let Σ be a ranked alphabet and H be a set. Prove or refute the following statements for every $\xi \in \mathrm{T}_\Sigma(H)$:

(a) $\forall w \in \mathsf{pos}(\xi)\colon \mathsf{pos}(\xi|_w) \subseteq \mathsf{pos}(\xi)$,

(b) $\forall w \in \mathsf{pos}(\xi)\colon \mathsf{sub}(\xi|_w) \subseteq \mathsf{sub}(\xi)$,

(c) $\forall w \in \mathsf{pos}(\xi), \zeta \in \mathrm{T}_\Sigma(H)\colon \mathsf{size}(\xi[\zeta]_w) = \mathsf{size}(\xi) + \mathsf{size}(\zeta) - \mathsf{size}(\xi|_w)$.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.

`http://www.orchid.inf.tu-dresden.de` `Tobias.Denkinger@tu-dresden.de`