Formale Baumsprachen

Task 4 (bu-det fta)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be ranked alphabets. Give deterministic bu-ta $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ that recognize L_1, L_2, and L_3, respectively, where

(a) $L_1 = \{\xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$,

(b) $L_2 = \{\xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$, and

(c) $L_3 = \{\sigma(t_1, \sigma(t_2, ...\sigma(t_n, \alpha)...)) \in T_\Delta \mid n \in \mathbb{N}, t_1, ..., t_n \in T_{\{\gamma^{(1)}, \alpha^{(0)}\}}\}$.

Task 5 (string automata I)

Recall the concept of string automata. Let Σ be an alphabet and $\# \notin \Sigma$. We define the ranked alphabet $\Sigma_\# = \Sigma_\#^{(0)} \cup \Sigma_\#^{(1)}$ where $\Sigma_\#^{(0)} = \{\#\}$ and $\Sigma_\#^{(1)} = \Sigma$. Moreover, we define the $\Sigma_\#$-algebra (Σ^*, θ) where $\theta(\#) = \varepsilon$ and $\theta(a)(w) = wa$ for every $a \in \Sigma$ and $w \in \Sigma^*$.

(a) Show that Σ^* is initial in the class of $\Sigma_\#$-algebras.

(b) We consider $\Sigma = \{a, b\}$ and the language $L = \{a^n b^m \mid n, m \in N\}$. Sketch the diagram of a total deterministic finite-state automaton accepting L and model the transition table using a finite $\Sigma_\#$-algebra Q. How can we interpret the uniquely determined homomorphism $h\colon \Sigma^* \to Q$?

(c) Convince yourself that any total deterministic finite-state automaton can be modeled as a quadruple $\mathcal{A} = (Q, \Sigma, \theta, F)$ where (Q, θ) is a finite $\Sigma_\#$-algebra and $F \subseteq Q$. Define the language accepted by $\mathcal{A}$ using the homomorphism $h\colon \Sigma^* \to Q$.

Task 6 (string automata II)

Let $\Sigma = \{\mathrm{a}, \mathrm{b}\}$ be an alphabet.

(a) Give a finite state automaton $\mathcal{A} = (Q, \Sigma, q_0, F)$ that recognizes

$$L = \{w \in \Sigma^* \mid |w|_\mathrm{a} - |w|_\mathrm{b} \bmod 2 \equiv 0\}.$$

(b) Describe L using a homomorphism between the free monoid $(\Sigma^*, \circ, \varepsilon)$ and the monoid $(\{0,1\}^{Q \times Q}, \times, 1_{Q \times Q})$.

(c) Describe L using a monoid with carrier $(\Sigma^*)^{Q \times Q}$.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.

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