

Formale Baumsprachen

Task 3 (universal algebra)

- (a) Recall the following concepts: Σ -algebra, Σ -homomorphism, initial Σ -algebra in a class \mathcal{K} , and Σ -term algebra.
- (b) Show that the mappings `height`, `size`, and `sub` (restricted to T_Σ) are homomorphisms. Start by giving the target algebra for each of them. What is the problem concerning `sub`?
- (c) Show that the principle of proof by structural induction is correct by applying the above concepts from universal algebra.

Task 4 (bu-det fta)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be ranked alphabets. Give deterministic bu-
ta \mathcal{A}_1 , \mathcal{A}_2 , and \mathcal{A}_3 that recognize L_1 , L_2 , and L_3 , respectively, where

- (a) $L_1 = \{\xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$,
- (b) $L_2 = \{\xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$, and
- (c) $L_3 = \{\sigma(t_1, \sigma(t_2, \dots \sigma(t_n, \alpha) \dots)) \in T_\Delta \mid n \in \mathbb{N}, t_1, \dots, t_n \in T_{\{\gamma^{(1)}, \alpha^{(0)}\}}\}$.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.