Formale Baumsprachen

Task 1 (definition by structural induction)

Let Σ be a ranked alphabet, $\xi, \xi_1, ..., \xi_k \in \mathrm{T}_\Sigma$, and $\zeta \in \mathrm{T}_\Sigma(X_k)$. Define the following functions by structural induction:

(a) $\mathsf{yield}(\xi)$, the sequence of leaves in ξ from left to right; and

(b) $\zeta[\xi_1, ..., \xi_k]$, the tree obtained from ζ by replacing every occurrence of x_i by ξ_i for every $i \in \{1, ..., k\}$.

In the lecture we defined trees as well-formed expressions. An alternative definition characterises a tree as a tuple (t, φ) where, intuitively, t is a set of *Gorn addresses* that is closed under certain operations and φ assigns a symbol from some alphabet Δ to every element of t.

(c) Give a formal definition of trees over Δ in the above sense.

Formally define the following characteristics of trees in the sense of Task 1 (c):

(d) height

(e) size

(f) set of positions

(g) set of subtrees

(h) label at a position

(i) subtree at a position

Task 2 (proof by structural induction)

Let A be a set, Σ be a ranked alphabet, $\xi, \zeta \in \mathrm{T}_\Sigma(A)$, and $w \in \mathsf{pos}(\xi)$. Prove or refute the following statements:

(a) $\xi(w) = \xi|_w(\varepsilon)$.

(b) $(\xi[\zeta]_w)|_w = \zeta$.

(c) $|\mathsf{pos}(\xi)| = |\mathsf{sub}(\xi)|$.

(d) $\mathsf{height}(\xi) = 1 + \max\{|\rho| \mid \rho \in \mathsf{pos}(\xi)\}$.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.

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