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# Formale Übersetzungsmodelle

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**Task 25** ( $TOP^R \subseteq d\text{-QREL}; TOP$ )

Let  $L_1, \dots, L_n \in \text{REC}(\Sigma)$  be recognizable tree languages,  $\mathcal{U} = \{0, 1\}^n$ , and  $\Sigma$  and  $\Omega$  be ranked alphabets where  $\Omega = \{ \langle \sigma, (u_1, \dots, u_k) \rangle^{(k)} \mid \sigma \in \Sigma, \text{rank}(\sigma) = k, u_1, \dots, u_k \in \mathcal{U} \}$ . Note that if  $k = 0$  we write  $\sigma$  rather than  $\langle \sigma, () \rangle$ . Consider the function  $B_{L_1, \dots, L_n} : T_\Sigma \rightarrow \mathcal{P}(T_\Omega)$  that is recursively defined for every  $\sigma \in \Sigma$  and  $t_1, \dots, t_k \in T_\Sigma$  by

$$B_{L_1, \dots, L_n}(\sigma(t_1, \dots, t_k)) = \{ \langle \sigma, (u_1, \dots, u_k) \rangle (t'_1, \dots, t'_k) \mid t'_1 \in B_{L_1, \dots, L_n}(t_1), \dots, t'_k \in B_{L_1, \dots, L_n}(t_k), \\ \forall i \in [k], \forall j \in [n]: u_i \in \mathcal{U}, u_i(j) = \text{if } t_i \in L_j \text{ then } 1 \text{ else } 0 \}$$

- (a) Give a deterministic state-relabeling bu-tt  $B$  that computes  $B_{L_1, \dots, L_n}$ .
- (b) Let  $\Sigma = \{ \sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)} \}$  be a ranked alphabet,  $n = 2$ , and  $L_1 = \{ \gamma^k(\beta) \mid k \in \mathbb{N} \}$  and  $L_2 = \{ \gamma^k(\alpha) \mid k \in \mathbb{N} \}$  be recognizable tree languages. Construct a deterministic state-relabeling bu-tt  $B'$  that computes  $B_{L_1, L_2}$ .

**Task 26** ( $l\text{-BOT} = l\text{-TOP}^R$ )

Consider the linear bu-tt  $B = (Q, \Sigma, \Sigma, F, R)$  where  $\Sigma = \{ \sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)} \}$ ,  $Q = \{ *, q, q_f \}$ ,  $F = \{ q_f \}$ , and  $R$  contains is given by

$$\begin{array}{lll} \alpha \rightarrow q(\alpha), & \alpha \rightarrow *( \alpha ), & \beta \rightarrow *( \beta ), \\ \gamma(q(x_1)) \rightarrow q(\gamma(x_1)), & \gamma(*(x_1)) \rightarrow *( \gamma(x_1) ), & \\ \sigma(*(x_1), q(x_2)) \rightarrow q_f(x_1), & \sigma(*(x_1), *(x_2)) \rightarrow *( \sigma(x_1, x_2) ) & \end{array}$$

- (a) Give a linear td-tt with regular look-ahead  $T$  such that  $\tau(T) = \tau(B)$ .
- (b) Construct a deterministic state-relabeling bu-tt  $B'$  and a linear td-tt  $T'$  such that  $\tau(T) = \tau(B') \circ \tau(T')$ .