Formale Übersetzungsmodelle

Task 21 (BOT^2 *and* TOP^2*)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the bu-tt $B = (Q_B, \Sigma, \Sigma, F, R_B)$ and the td-tt $T = (Q_T, \Sigma, \Sigma, I, R_T)$ where $Q_B = \{*, q, q_f\}$, $F = \{q_f\}$, $Q_T = \{*, q\}$, $I = \{*\}$, and

$$\begin{aligned} R_B = \{\ & \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2)), \\ & \sigma(*(x_1), q(x_2)) \to q_f(x_1), \\ & \gamma(*(x_1)) \to *(\gamma(x_1)), \\ & \gamma(q(x_1)) \to q(\gamma(x_1)), \\ & \gamma(q_f(x_1)) \to q_f(\gamma(x_1)), \\ & \alpha \to *(\alpha),\ \ \alpha \to q(\alpha),\ \ \beta \to q(\beta)\ \ \} \end{aligned}$$

$$\begin{aligned} R_T = \{\ & q(\sigma(x_1, x_2)) \to \sigma(q(x_1), q(x_2)), \\ & *(\sigma(x_1, x_2)) \to \sigma(q(x_1), *(x_1)), \\ & *(\sigma(x_1, x_2)) \to \sigma(*(x_1), q(x_1)), \\ & *(\gamma(x_1)) \to \gamma(*(x_1)), \\ & q(\gamma(x_1)) \to \gamma(q(x_1)), \\ & *(\alpha) \to \alpha,\ \ q(\alpha) \to \alpha,\ \ *(\beta) \to \beta\ \ \} \end{aligned}$$

(a) Identify the bottom-up and top-down specific properties of $\tau(B)$ and $\tau(T)$.

(b) Give td-tt T_1 and T_2 and bu-tt B_1 and B_2 such that $\tau(B) = \tau(T_1); \tau(T_2)$ and $\tau(T) = \tau(B_1); \tau(B_2)$.

Task 22 (Baker's theorem for BOT*)*

Theorem [Bak79, Thm. 6]. Let B_1 and B_2 be bu-tt. Then $\tau(B_1); \tau(B_2) \in \mathsf{BOT}$ if the following two conditions hold:

1. B_1 is linear or B_2 is deterministic;
2. B_1 is nondeleting or B_2 is total.

(a) Give two bu-tt B_1' and B_2' that fulfill Condition 1 but not Condition 2 such that $\tau(B_1'); \tau(B_2') \notin \mathsf{BOT}$. Give two bu-tt B_1'' and B_2'' that do not fulfill Condition 1 but fulfill Condition 2 such that $\tau(B_1''); \tau(B_2'') \notin \mathsf{BOT}$. For each bu-tt, use the minimum number of rules necessary.

(b) Construct the instance B' and B'' (for B_1' and B_2', and B_1'' and B_2'', respectively) of the bu-tt B defined in the proof (from the lecture) of the above theorem.

(c) Give trees s', t', s'', t'' such that

  (i) $\neg((s', t') \in \tau(B_1') \circ \tau(B_2') \iff (s', t') \in \tau(B'))$ and

  (ii) $\neg((s'', t'') \in \tau(B_1'') \circ \tau(B_2'') \iff (s'', t'') \in \tau(B''))$.

(d) Prove the following corollary:

> **Corollary**. Let B_1 and B_2 be bu-tt. Then $\tau(B_1); \tau(B_2) \in \mathsf{BOT}$ if B_1 is linear or B_2 is deterministic.

(e) Apply the above corollary to B_1' and B_2' from Task 22 (a).

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References

[Bak79] Brenda S. Baker. Composition of top-down and bottom-up tree transductions. *Information and Control*, 41(2):186–213, 1979.

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