

## Formale Übersetzungsmodelle

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### Task 15 (*Symbolic derivation*)

Let  $k \in \mathbb{N}$  and  $\Sigma = \{+, \cdot, -, \sin, \cos, X^{(0)}\} \cup \{0^{(0)}, \dots, k^{(0)}\}$  a ranked alphabet. The trees over  $\Sigma$  represent a subset of polynomials with natural number coefficients.

- (a) Give a td-tt  $T$  that computes the symbolic derivation of a given tree.
- (b) Give a derivation for  $\xi = \cdot(+(\sin(\cdot(X, X)), 5), X)$  in  $T$ .
- (c) Why is there no bu-tt  $B'$  such that  $\tau(T) = \tau(B')$ ?
- (d) Give a bu-tt  $B$  that simplifies a given tree according to the units with respect to  $\cdot$  and  $+$  and the absorbing nature of 0 with respect to  $\cdot$ , in particular,  $B$  should collapse  $\cdot(\xi, 1)$ ,  $\cdot(1, \xi)$ ,  $+(\xi, 0)$ , or  $+(0, \xi)$  to  $\xi$  and  $\cdot(\xi, 0)$  or  $\cdot(0, \xi)$  to 0 for every  $\xi \in T_\Sigma$ .
- (e) Why is there no td-tt  $T'$  such that  $\tau(B) = \tau(T')$ ?