4. Übung (10. November 2016)

Formale Übersetzungsmodelle

Task 9 (powerset construction)

Let $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$ be a ranked alphabet. Consider the bottom-up finite state tree automaton $N = (\{q_0, q_1\}, \Sigma, \Sigma, \{q_0\}, R)$ where R is given by

$$\alpha \to q_0(\alpha) \qquad \text{and} \qquad \sigma(q_i(x_1), q_j(x_2)) \to q_{1-k}(\sigma(x_1, x_2))$$

for every $i, j \in \{0, 1\}$ and $k \in \{i, j\}$.

(a) Determine the tree language of N.

(b) Use the powerset construction to give a deterministic bottom-up finite state tree automaton N_{det} such that $\tau(N) = \tau(N_{\text{det}})$.

Task 10 (bounded growth property)

Prove the following statement [cf. Eng75, Lem. 1.1, p. 205]:

Claim. There exists a $c \in \mathbb{N}$ such that for every $(s, t) \in \tau(M)$ holds $\mathsf{height}(t) \le c \cdot \mathsf{height}(s)$.

LEMMA 1.1. *Let $B = \langle \Sigma, \Delta, Q, Q_d, R \rangle$ be a bottom-up fst.*

(1) *For $\sigma \in \Sigma_0$, $q \in Q$ and $s \in T_\Delta$, if $\sigma \overset{*}{\Rightarrow} q(s)$, then the rule $\sigma \to q(s)$ is in R.*

(2) *For $k \ge 1$, $\sigma \in \Sigma_k$, $t_1, \cdots, t_k \in T_\Sigma$, $q \in Q$ and $s \in T_\Delta$, if $\sigma(t_1 \cdots t_k) \overset{*}{\Rightarrow} q(s)$, then there exist states $q_1, \cdots, q_k$ and trees $s_1, \cdots, s_k$ in T_Δ such that $\sigma(t_1 \cdots t_k) \overset{*}{\Rightarrow} \sigma(q_1(s_1) \cdots q_k(s_k)) \Rightarrow q(s)$ and for all i $(1 \le i \le k)$ $t_i \overset{*}{\Rightarrow} q_i(s_i)$.*

(3) *For $k \ge 1$, $t_1, \cdots, t_k \in T_\Sigma$, $q_1, \cdots, q_k \in Q$, $s_1, \cdots, s_k \in T_\Delta$ and $\sigma \in \Sigma_k$, if for all i $(1 \le i \le k)$ $t_i \overset{*}{\Rightarrow} q_i(s_i)$, then $\sigma(t_1 \cdots t_k) \overset{*}{\Rightarrow} \sigma(q_1(s_1) \cdots q_k(s_k))$.*

Task 11 ($\mathsf{BOT} \subseteq \mathsf{REL} \,;\, \mathsf{FTA} \,;\, \mathsf{HOM}$*)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$. Consider the bu-tt $M = (\{q_0, q_1, p\}, \Sigma, \Sigma, \{q_0\}, R)$, where R is given by:

$$\alpha \to q_0(\alpha), \qquad \gamma(p(x_1)) \to p(\gamma(x_1)), \qquad \sigma(q_0(x_1), p(x_2)) \to q_1(\sigma(x_2, x_1)),$$
$$\alpha \to p(\alpha), \qquad \sigma(q_1(x_1), p(x_2)) \to q_0(\sigma(x_1, x_1)).$$

(a) Describe $\tau(M)$.

(b) Construct, according to the decomposition result from the lecture, a relabeling bu-tt M_1, an fta M_2, and a homomorphism bu-tt M_3 such that $\tau(M) = \tau(M_1) \,;\, \tau(M_2) \,;\, \tau(M_3)$.

(c) Illustrate the transformation $\tau(M_1) \,;\, \tau(M_2) \,;\, \tau(M_3)$ with the input tree $\sigma(\alpha, \gamma\alpha) \in T_\Sigma$.

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Task 12 ***(*** $\mathsf{BOT} \subseteq \mathrm{QREL} \,;\mathrm{HOM}$***)***

(a) Elaborate on the construction of the qrel bu-tt in the proof of $\mathsf{BOT} \subseteq \mathsf{QREL} \,;\mathsf{HOM}$.

(b) Apply the above construction to the bu-tt $M = (\{q_0, q_1, p\}, \Sigma, \Sigma, \{q_0\}, R)$, where $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and R is given by:

$$\begin{aligned}
&\alpha \to q_0(\alpha), \qquad && \gamma(p(x_1)) \to p(\gamma(x_1)), \qquad && \sigma(q_0(x_1), p(x_2)) \to q_1(\sigma(x_2, x_1)),\\
&\alpha \to p(\alpha), && && \sigma(q_1(x_1), p(x_2)) \to q_0(\sigma(x_1, x_1)).
\end{aligned}$$

I.e. give a qrel bu-tt M_1' and a homomorphism bu-tt M_2' such that $\tau(M) = \tau(M_1') \,; \tau(M_2')$.

References

[Eng75] J. Engelfriet. “Bottom-up and top-down tree transformations—a comparison”. In: *Mathematical systems theory* 9.2 (1975), pp. 198–231. issn: 0025-5661. doi: `10.1007/BF01704020`.

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