Maschinelles Übersetzen natürlicher Sprachen

3. Übungsblatt

2016-11-10

Aufgabe 1

Let $n \in \mathbb{N}$, and let $\mathbb{W}$, $\mathbb{X}$, $\mathbb{Y}$, and $\mathbb{Z}$ be random variables. Show the following implication. If

$$P(\mathbb{W} = w \mid \mathbb{X} = x, \mathbb{Y} = y, \mathbb{Z} = z) = P(\mathbb{W} = w \mid \mathbb{Z} = z) \quad \text{for every } w, x, y, z,$$

then

$$P(\mathbb{W} = w \mid \mathbb{Y} = y, \mathbb{Z} = z) = P(\mathbb{W} = w \mid \mathbb{Z} = z) \quad \text{for every } w, y, z.$$

Aufgabe 2

Let $H = (Q, V, \#, t, e)$ be a hidden Markov model and $v_1 \dots v_n \in V^+$ for some $n \geq 1$.

1. Derive the backward algorithm from the formal definition of HMM by giving a closed and a recursive definition of $S(i, q)$ where $1 \leq i \leq n$ and $q \in Q$. Show that

$$S(i, q) = P(\mathbb{V}_{i+1} = v_{i+1}, \dots, \mathbb{V}_n = v_n, \mathbb{L} = n \mid \mathbb{Q}_i = q).$$

2. Derive the forward algorithm from the formal definition of HMM by giving a closed and a recursive definition of $T(i, q)$ where $1 \leq i \leq n$ and $q \in Q$. Show that

$$T(i, q) = P(\mathbb{V}_1 = v_1, \dots, \mathbb{V}_i = v_i \mid \mathbb{Q}_i = q).$$

Aufgabe 3

Let $V_E = \{a, b\}$ and $V_F = \{\alpha, \beta, \gamma\}$ be an English and French vocabulary, respectively. Consider the following bigram model, length model, and dictionary.

$b(\rightarrow \mid \downarrow)$	$\#$	a	b
$\#$	2^{-1}	2^{-1}	0
a	0	2^{-1}	2^{-1}
b	2^{-1}	0	2^{-1}

$$\varepsilon(m \mid l) = \begin{cases} 2^{-1} & \text{if } m = l \\ 2^{-2} & \text{if } |m - l| = 1 \\ 0 & \text{otherwise} \end{cases}$$

$t(\rightarrow \mid \downarrow)$	α	β	γ
a	2^{-1}	0	2^{-1}
b	0	2^{-1}	2^{-1}

Decode the sentence $\beta\gamma$ using the algorithm from the lecture. Annotate the hypotheses with a target length to make the calculation feasible.

Zusatzaufgabe 1

Write a small program to solve the previous task.