# Formale Baumsprachen

### *Task 27 (for every M-expression there is an equivalent M-wta)*

Consider the ranked alphabet $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and the M-monoid $(\mathcal{P}(\mathrm{T}_\Sigma), \cup, \emptyset, \Omega_\Sigma)$ where $\Omega_\Sigma = \{\emptyset_k \mid k \in \mathbb{N}\} \cup \{\mathsf{top}_\delta \mid \delta \in \Sigma\} \cup \{\mathsf{proj}_1^{(2)}\}$, $\mathsf{proj}_1^{(2)} \in \mathsf{Ops}^2(\mathcal{P}(\mathrm{T}_\Sigma))$ such that

$$\forall L_1, L_2 \in \mathcal{P}(\mathrm{T}_\Sigma): \qquad \mathsf{proj}_1^{(2)}(L_1, L_2) = L_1,$$

and for every $k \in \mathbb{N}$ and $\delta \in \Sigma^{(k)}$ we have that $\emptyset_k, \mathsf{top}_\delta \in \mathsf{Ops}^k(\mathcal{P}(\mathrm{T}_\Sigma))$ such that

$$\forall L_1, \ldots, L_k \in \mathcal{P}(\mathrm{T}_\Sigma): \qquad \begin{aligned} \emptyset_k(L_1, \ldots, L_k) &= \emptyset, \text{ and} \\ \mathsf{top}_\delta(L_1, \ldots, L_k) &= \{\delta(\xi_1, \ldots, \xi_k) \mid \xi_1 \in L_1, \ldots, \xi_k \in L_k\}. \end{aligned}$$

Using the construction from the lecture, give an M-automaton that is equivalent to the M-expression

$$e_\tau = H(\omega) + \textstyle\sum_x \mathsf{label}_\sigma(x) \rhd H(\omega')$$

where

$$\omega = (\omega_{(\delta,\emptyset)} \mid (\delta,\emptyset) \in \Sigma_\emptyset) \qquad \omega_{(\delta,\emptyset)} = \mathsf{top}_\delta, \text{ and}$$

$$\omega' = (w'_{(\delta,V)} \mid (\delta, V) \in \Sigma_{\{x\}}) \qquad \omega'_{(\delta,V)} = \begin{cases} \mathsf{proj}_1^{(2)} & \text{if } (\delta, V) = (\sigma, \{x\}), \\ \mathsf{top}_\delta & \text{if } (\delta, V) \neq (\sigma, \{x\}). \end{cases}$$


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