Formale Baumsprachen

Task 21 (zigzag is not bottom-up deterministically recognizable)

Recall the arctic semiring $\mathsf{Arct} = (\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$. Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ be a ranked alphabet and $\mathsf{zigzag}\colon \mathrm{T}_\Sigma \to \mathbb{N}$ a mapping such that

$$\begin{aligned}\mathsf{zigzag}(\alpha) &= 1\\ \mathsf{zigzag}(\sigma(\alpha, \xi_1)) &= 2\\ \mathsf{zigzag}(\sigma(\sigma(\xi_1, \xi_2), \xi_3)) &= 2 + \mathsf{zigzag}(\xi_2)\end{aligned}$$

(b) Show that zigzag is *not* bottom-up deterministically recognizable over the arctic semiring.

Task 22 (representation lemma and implementation lemma)

Let A be a set and $\mathcal{M} = (\mathcal{P}(\Sigma), \Sigma, S, \delta, F)$ an S-weighted tree automaton such that $\Sigma = \{\sigma^{(2)}, \gamma^{(\gamma)}, \alpha^{(0)}, \beta^{(0)}\}$, $S = (\mathcal{P}(A), \cup, \cap, \emptyset, A)$, $F(p) = A$ for every $p \in \mathcal{P}(\Sigma)$, and

$$\delta_\tau(q_1 \ldots q_{\mathrm{rk}(\tau)}, q) = \begin{cases} A & \text{if } q \subseteq (\bigcup q_1) \cup \ldots \cup (\bigcup q_{\mathrm{rk}(\tau)}) \cup \{\tau\},\\ \emptyset & \text{otherwise.}\end{cases} \qquad \text{for every } \tau \in \Sigma$$

(a) Is S locally finite?

(b) *Representation lemma.* Give a finite Σ-algebra over Q as in the proof from the lecture and a mapping $f\colon Q \to S$ such that $r_\mathcal{M} = f \circ h_Q$.

(c) *Implementation lemma.* Show that $f \circ h_Q \in \text{bud-Rec}(\Sigma, S)$.

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