Formale Baumsprachen

Task 19 (closure of under multiplication with a scalar)

Show that semiring-weighted recognizable tree languages are closed under left and right multiplication with a scalar.

Task 20 (analysis of weighted tree automata)

Consider the S-weighted tree automaton $\mathcal{A} = (Q, \Sigma, S, \delta, F)$ be an S-weighted tree automaton where $Q = \{q_1, q_2\}$, $\Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{s^{(0)} \mid s \in S\}$, $(S, +, \cdot, 0, 1)$ is a finite semiring, $F(q_1) = 1$, $F(q_2) = 0$, and all non-zero assignments in δ are given by

$$\begin{aligned} &\delta_s(\varepsilon, q_1) = s && \delta_s(\varepsilon, q_2) = 1 && \text{for every } s \in S \\ &\delta_\oplus(q_1 q_2, q_1) = 1 && \delta_\oplus(q_2 q_1, q_1) = 1 && \delta_\oplus(q_2 q_2, q_2) = 1 \\ &\delta_\odot(q_1 q_1, q_1) = 1 && \delta_\odot(q_2 q_2, q_2) = 1. \end{aligned}$$

Describe the series induced by $\mathcal{A}$. Illustrate your observation with an example.

Task 21 (zigzag is not bottom-up deterministically recognizable)

Recall the arctic semiring $\mathsf{Arct} = (\mathbb{R} \cup \{-\infty\}, \mathsf{max}, +, -\infty, 0)$. Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ be a ranked alphabet and $\mathsf{zigzag}\colon \mathrm{T}_\Sigma \to \mathbb{N}$ a mapping such that

$$\begin{aligned} \mathsf{zigzag}(\alpha) &= 1 \\ \mathsf{zigzag}(\sigma(\alpha, \xi_1)) &= 2 \\ \mathsf{zigzag}(\sigma(\sigma(\xi_1, \xi_2), \xi_3)) &= 2 + \mathsf{zigzag}(\xi_2) \end{aligned}$$

(a) Show that zigzag is recognizable over the arctic semiring.

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