Formale Baumsprachen

Task 17 (properties of some algebras)

Characterize the algebraic properties of the following algebras

(a) $\mathsf{Prob} = (\mathbb{R}_{\geq 0}, +, \cdot, 0, 1)$

(b) $\mathsf{Vit} = ([0,1], \max, \cdot, 0, 1)$

(c) $\mathsf{Fuzzy} = ([0,1], \max, \min, 0, 1)$

(d) $\mathsf{Prob}_1 = ([0,1], \oplus_1, \cdot, 0, 1)$

(e) $\mathsf{Prob}_2 = ([0,1], \oplus_2, \cdot, 0, 1)$

(f) $\mathsf{DivLat} = (\mathbb{N}, \mathrm{lcm}, \gcd, 1, 0)$

(g) $\mathsf{Trop} = (\mathbb{R} \cup \{\infty\}, \min, +, \infty, 0)$

(h) $\mathsf{Arct} = (\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$

(i) $\mathsf{Trop}' = (\mathbb{R} \cup \{\infty\}, +, \min, 0, \infty)$

(j) $\mathsf{Arct}' = (\mathbb{R} \cup \{-\infty\}, +, \max, 0, -\infty)$

(k) $\mathsf{Pow}_A = (\mathcal{P}(A), \cup, \cap, \emptyset, A)$

(l) $\mathsf{Lang}_\Sigma = (\mathcal{P}(\Sigma^*), \cup, \circ, \emptyset, \{\varepsilon\})$

(m) $\mathsf{Lang}'_\Sigma = (\mathcal{P}(\Sigma^*), \cap, \circ, \Sigma^*, \{\varepsilon\})$

(n) $\mathsf{PrefLat}_\Sigma = (\Sigma^* \cup \{\top\}, \wedge, \circ, \top, \varepsilon)$

(o) $\mathsf{TLang}_\Delta = (\mathcal{P}(\mathrm{T}_\Delta), \cup, \cdot_\alpha, \emptyset, \{\alpha\})$

where $p_1 \oplus_1 p_2 = p_1 + p_2 - p_1 \cdot p_2$ and $p_1 \oplus_2 p_2 = \min\{p_1 + p_2, 1\}$ for every $p_1, p_2 \in [0,1]$, A is an arbitrary set, Σ is an alphabet, $\circ$ is the concatenation, $\wedge$ is the longest common prefix operator, $\top$ is neutral with respect to $\wedge$ and annihilating with respect to $\cdot$, Δ is a ranked alphabet, $\alpha \in \Delta^{(0)}$, and $\cdot_\alpha$ is the α-concatenation.

Task 18 (calculating tree characteristics with weighted tree automata)

(a) Let Σ be a ranked alphabet and $e \in \Sigma^{(0)}$. Give a weighted tree automaton $\mathcal{A}$ over a suitable strong bimonoid such that $\mathrm{r}_\mathcal{A} \cong \mathsf{yield}$.

(b) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Give a weighted tree automaton $\mathcal{B}$ over a suitable semiring such that $\mathrm{r}_\mathcal{B} = \mathsf{pos}$. Draw $\mathcal{B}$ as a hypergraph.

(c) Let Σ be a ranked alphabet. Give a weighted tree automaton $\mathcal{C}$ over a suitable semiring such that $\mathrm{r}_\mathcal{C} = \mathsf{height}$.

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