Formale Baumsprachen

Task 15 (Rec = *MSO-definable)*

(a) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Consider the bottom-up deterministic fta $\mathcal{A} = (Q, \Sigma, \delta, F)$ with $Q = \{0, 1\}$, $F = \{1\}$, and $\delta_\sigma(q_1, q_2) = \mathsf{max}\{q_1, q_2\}$ for every $q_1, q_2 \in Q$, $\delta_\gamma(q) = 1$ for every $q \in Q$, and $\delta_\alpha() = 0$. Use the construction from the lecture to show that $L(\mathcal{A})$ is MSO-definable.

(b) Recall the following Lemma from the lecture:

Lemma. Let Σ be a ranked alphabet and $\mathcal{V} \subseteq_{\text{fin}} \mathcal{V}_1$. Then $T^{\text{v}}_{\Sigma_\mathcal{V}}$ is recognizable.

In the proof we required a family of languages $(L_x \mid x \in \mathcal{V})$ where for every $x \in \mathcal{V}$:

$$L_x = \{\xi \in T_{\Sigma_\mathcal{V}} \mid x \text{ occurs exactly once in } \xi\}.$$

Construct an automaton $\mathcal{A}_x$ for every $x \in \mathcal{V}$ such that $L(\mathcal{A}_x) = L_x$.

(c) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\varphi = \exists x.\mathsf{label}_\gamma(x)$. Use the construction from the lecture to show that $L(\varphi)$ is recognizable.

Task 16 (pumping lemma for Rec*)*

Prove the following lemma.

Lemma. Let Σ be a ranked alphabet and $L \in \mathsf{Rec}(\Sigma)$. Then there is a $p \in \mathbb{N}$ such that for every $\xi \in L$, the following implication holds:

If $\mathsf{height}(\xi) \geq p$, then there are $u, v \in \mathsf{C}_{\Sigma,1}$ and $w \in \mathsf{T}_\Sigma$ such that

(i) $\xi = u[v[w]]$,

(ii) $\mathsf{height}(v[w]) \leq p$,

(iii) $\mathsf{height}(v) \geq 2$, and

(iv) for every $n \in \mathbb{N}$, $u[v^n[w]] \in L$.

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