5. Übung (May 11, 2016)

Formale Baumsprachen

Task 13 (Myhill-Nerode theorem for trees)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $L \subseteq T_\Sigma$ be the language consisting of all trees with exactly as many αs as βs. Use the Myhill-Nerode theorem to show that L is not recognizable.

Task 14 (monadic second-order logic on trees)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the MSO-formula

$$\varphi = \exists U.\neg\exists x.\exists y.\mathsf{edge}_2(x, y) \wedge \mathsf{label}_\sigma(y) \wedge x \in U$$

over Σ where $x, y \in \mathcal{V}_1$ and $U \in \mathcal{V}_2$.

(a) Calculate $\mathtt{Fr}(\varphi)$ and $\mathtt{Bd}(\varphi)$ using the definitions from the lecture.

(b) Is φ closed?

Consider the tree $\xi = \sigma(\gamma(\alpha), \beta)$ and the following functions:

$$\begin{aligned}
\rho_1&\colon\ x \mapsto \varepsilon,\ \ x' \mapsto 1,\ \ y \mapsto 11,\ \ y' \mapsto 2,\\
\rho_2&\colon\ x \mapsto \varepsilon,\ \ x' \mapsto \varepsilon,\ \ \bar{x} \mapsto 1,\ \ y \mapsto 11,\ \ y' \mapsto 2,\\
\rho_3&\colon\ X \mapsto \{\varepsilon, 1\},\ \ Y \mapsto \{11, 2\}, \text{ and}\\
\rho_4&\colon\ X \mapsto \emptyset,\ \ Y \mapsto \{1, 2, 3\},\ \ x \mapsto \varepsilon.
\end{aligned}$$

(c) Which of the functions $\rho_1, ..., \rho_4$ are assignments for ξ? Give the appropriate sets of variables.

(d) Encode the assignments from Task 14 (c) as trees.

(e) Which of the trees obtained in Task 14 (d) are valid?

Let $\mathcal{V} = \{x, y, U\}$. Construct MSO-formulas φ_1, φ_2, and φ_3 such that $\mathtt{Fr}(\varphi_1), \mathtt{Fr}(\varphi_2), \mathtt{Fr}(\varphi_3) \subseteq \mathcal{V}$ and for every $\xi \in T_\Sigma$ and $\mathcal{V}$-assignment ρ for ξ:

(f) $(\xi, \rho) \models \varphi_1$ iff there is a downward path from the node $\rho(x)$ to the node $\rho(y)$ in ξ, i.e. there is a $w \in \mathbb{N}^*$ such that $\rho(y) = \rho(x)w$.

(g) $(\xi, \rho) \models \varphi_2$ iff $\rho(U)$ is the set of all positions w in ξ such that $\xi|_w = \sigma(\alpha, \beta)$.

(h) $(\xi, \rho) \models \varphi_3$ iff for every node in ξ labeled by σ, none of its child nodes is labeled by γ.