Formale Baumsprachen

Task 10 (relabelings)

(a) Show that any relabeling preserves the image under pos.

Let Σ and Δ be ranked alphabets.

(b) Under which conditions is there a relabeling between trees over Σ and trees over Δ?

(c) Let τ be a relabeling between trees over Σ and trees over Δ. Now consider $\sigma \in \Sigma$, $\xi \in T_\Sigma$, and $L \subseteq T_\Sigma$. Quantify τ in the following expressions:

(i) $\tau(\sigma)$, (ii) $\tau(\xi)$, and (iii) $\tau(L)$

Task 11 (construction of Bar-Hillel, Perles, and Shamir)

Consider the ranked alphabet $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)}\}$ and the fta $\mathcal{A} = (Q, \Sigma, \delta, F)$ where $Q = \{e, o\}$, $F = \{e\}$, and

$$\delta_\alpha = \delta_\beta = \delta_\gamma = \{(\varepsilon, o)\}, \qquad \delta_\sigma = \{(q_1 q_2, q_0) \in Q^2 \times Q \mid q_0 = o \text{ iff } q_1 = q_2\}.$$

Moreover, let us assume an fsa $\mathcal{B} = (P, \Delta, p, \mu, G)$ where $\Delta = \Sigma^{(0)} \setminus \{\lambda\}$, $P = \{p, r\}$, $G = \{r\}$, and

$$\mu = \{(p, \alpha, p), (p, \beta, r), (r, \beta, r)\}.$$

Using the technique from the lecture, construct an fta $\mathcal{A}'$ such that

$$L(\mathcal{A}') = L(\mathcal{A}) \cap \mathsf{yield}_\lambda^{-1}(L(\mathcal{B})).$$

Task 12 (construction for Rec $\subseteq$ Rat*)*

Consider the ranked alphabet $\Sigma = \{\alpha^{(0)}, \gamma^{(1)}\}$.

(a) Give sets N and P such that the regular tree grammar $G = (N, \Sigma, Z, P)$ recognizes

$$L = \{\xi \in T_\Sigma \mid \text{the number of occurrences of } \gamma \text{ in } \xi \text{ is } \textit{not} \text{ divisible by } 3\}\ .$$

(b) Convince yourself that $L^N_{Z,\emptyset} = L$ using the following definition and property:

Definition. For every $Q, K \subseteq N$ such that $Q \cap K = \emptyset$, and for every $A \in N$:

$$L^Q_{A,K} = \{\xi \in T_\Sigma(K) \mid \text{there is a derivation } A \Rightarrow_G \xi_1 \Rightarrow_G \dots \Rightarrow_G \xi_n \Rightarrow_G \xi_{n+1} = \xi \text{ with } n \geq 0 \text{ such that for every } i \in [n]\text{: } \xi_i \in T_\Sigma(Q \cup K) \text{ and a rule with left-hand side in } Q \text{ is applied to } \xi_i \text{ to obtain } \xi_{i+1}\}$$

Property. For every $Q, K \subseteq N$ and $A, B \in N$ such that $B \in N \setminus Q$ and $(Q \cup \{B\}) \cap K = \emptyset$:

$$L^{Q \cup \{B\}}_{A,K} = L^Q_{A,K \cup \{B\}} \cdot_B \left(L^Q_{B,K \cup \{B\}}\right)^*_B \cdot_B L^Q_{B,K}$$

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