Formale Baumsprachen

Task 7 (closure of Rec *under intersection, union, and complement)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the following recognizable tree languages

$$L_1 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathsf{pos}(\xi)\colon w \in \{2\}^* \text{ if and only if } \xi(w) \in \{\sigma, \alpha\}\} \text{ and}$$
$$L_2 = \{\xi \in T_\Sigma \mid \text{for every } w \in \mathsf{pos}(\xi)\colon \xi(w) = \alpha \text{ only if } |w| \equiv 0 \pmod 2\}.$$

Find finite representations for the following languages:

(a) L_1 (b) L_2 (c) $L_1 \cup L_2$ (d) $L_1 \cap L_2$ (e) $T_\Sigma \setminus L_1$

Task 8 (concatenation and Kleene star for recognizable tree languages)

Let Σ be a ranked alphabet.

(a) Show that $\mathsf{Rec}(\Sigma)$ is closed under top concatenation without using the fact that it is closed under tree concatenation.

(b) Why can we not use the closure of $\mathsf{Rec}(\Sigma)$ under tree concatenation to prove the closure under Kleene star?

Prove or refute the following two statements:

(c) For every $\alpha \in \Sigma^{(0)}$, the binary operation $\cdot_\alpha$ is associative.

(d) $(L_1 \cdot_\alpha L_2) \cdot_\beta L_3 = L_1 \cdot_\alpha (L_2 \cdot_\beta L_3)$ for arbitrary $L_1, L_2, L_3 \in \mathsf{Rec}(\Sigma)$ and $\alpha, \beta \in \Sigma^{(0)}$.

Let $\Delta = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet.

(e) Using the construction from the lecture, show that $\{\sigma(\alpha, \beta)\}^*_\beta \cdot_\beta \{\alpha\} \in \mathsf{Rec}(\Sigma)$.

Task 9 (finite state automata)

Let $\Sigma = \{\mathrm{a}, \mathrm{b}\}$ be an alphabet.

(a) Give a finite state automaton $\mathcal{A} = (Q, \Sigma, q_0, F)$ that recognizes

$$L = \{w \in \Sigma^* \mid |w|_\mathrm{a} - |w|_\mathrm{b} \bmod 2 \equiv 0\}.$$

(b) Describe L using a homomorphism between the free monoid $(\Sigma^*, \circ, \varepsilon)$ and the monoid $(\{0,1\}^{Q\times Q}, \times, 1_{Q\times Q})$.

(c) Describe L using a monoid with carrier $(\Sigma^*)^{Q\times Q}$.

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