Ergänzungen zum maschinellen Übersetzen natürlicher Sprachen

4. Übungsblatt

2016-05-24

Exercise 1

Let $n \in \mathbb{N}$. Let $\Omega_1, \ldots, \Omega_n$ and $X_1, \ldots, X_n$ be non-empty, finite sets. For every $i \in \{1, \ldots, n\}$, let $p^{(i)} : \Omega_i \to \mathcal{M}(X_i)$ be a Ω_i-probability model. Let $\Omega = \Omega_1 \times \ldots \times \Omega_n$ and $X = X_1 \times \ldots \times X_n$. We define the Ω-probability model $p : \Omega \to \mathcal{M}(X)$ by

$$p_{(\omega_1,\ldots,\omega_n)}(x_1, \ldots, x_n) = p^{(1)}_{\omega_1}(x_1) \cdot \ldots \cdot p^{(n)}_{\omega_n}(x_n)$$

for every $\omega_i \in \Omega_i$, $x_i \in X_i$, $i \in \{1, \ldots, n\}$.

Let c be an X-corpus. For every $i \in \{1, \ldots, n\}$, let c_i be an X_i-corpus such that $c_i(x') = \sum_{x=(x_1,\ldots,x_n)\in X\colon x_i = x'} c(x)$. Show that $\mathrm{mle}_p(c) = \mathrm{mle}_{p^{(1)}}(c_1) \times \ldots \times \mathrm{mle}_{p^{(n)}}(c_n)$.

Exercise 2

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet, $G = (Q, Z, R)$ be an RTG with $Q = \{Z, A, B\}$, and $p : R \to \mathbb{R}_{\geq 0}$, where the rules in R and their probabilities are listed below:

$$\begin{aligned}
Z &\to \sigma(A,B) && \# \ 0.4\\
Z &\to \gamma(A) && \# \ 0.6\\
A &\to \alpha && \# \ 0.5\\
A &\to \gamma(A) && \# \ 0.5\\
B &\to \beta && \# \ 0.6\\
B &\to \gamma(Z) && \# \ 0.4
\end{aligned}$$

1. Compute the inside weight $\beta(q)$ for each $q \in Q$ by computing the fixpoint of an appropriate mapping F. Specify F in advance.
2. Afterwards, compute the outside weight $\alpha(q)$ for each $q \in Q$. Note that $\alpha(q)$ can also be defined recursively as follows:

$$\begin{aligned}
\alpha(q) &= \delta_{q_0}(q)\\
&\quad + \sum_{i,q_1,\ldots,q_k,\sigma} \alpha(q_i) \cdot p(q_i)(q_1 \ldots q_{i-1} q q_{i+1} \ldots q_k, \sigma) \cdot \beta(q_1) \cdot \ldots \cdot \beta(q_{i-1}) \cdot \beta(q_{i+1}) \cdot \ldots \cdot \beta(q_k)
\end{aligned}$$

   where $\delta_{q_0}(q) = 1$ if $q = q_0$ and 0 otherwise.

Exercise 3

Let $q \in [0, 1]$ and let (G, p) be a probabilistic RTG with initial symbol S and the following rules and probabilities:

$$\begin{aligned}
S &\to \sigma(S,S) && \# \ q\\
S &\to \alpha && \# \ 1-q
\end{aligned}$$

Show that $\beta(S) = 1$ iff $q \leq 0.5$.