## Formale Übersetzungsmodelle

## Task 21 (BOT<sup>2</sup> and TOP<sup>2</sup>)

Let  $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$  be a ranked alphabet. Consider the bu-tt  $B = (Q_B, \Sigma, \Sigma, F, R_B)$  and the td-tt  $T = (Q_T, \Sigma, \Sigma, I, R_T)$  where  $Q_B = \{*, q, q_f\}, F = \{q_f\}, Q_T = \{*, q\}, I = \{*\}, A$  and

$$\begin{split} R_B &= \{ \begin{array}{ll} \sigma(*(x_1),*(x_2)) \rightarrow *(\sigma(x_1,x_2)), & R_T &= \{ \begin{array}{ll} q(\sigma(x_1,x_2)) \rightarrow \sigma(q(x_1),q(x_2)), \\ \sigma(*(x_1),q(x_2)) \rightarrow q_f(x_1), & *(\sigma(x_1,x_2)) \rightarrow \sigma(q(x_1),*(x_1)), \\ \gamma(*(x_1)) \rightarrow *(\gamma(x_1)), & *(\sigma(x_1,x_2)) \rightarrow \sigma(*(x_1),q(x_1)), \\ \gamma(q(x_1)) \rightarrow q(\gamma(x_1)), & *(\gamma(x_1)) \rightarrow \gamma(*(x_1)), \\ \gamma(q_f(x_1)) \rightarrow q_f(\gamma(x_1)), & q(\gamma(x_1)) \rightarrow \gamma(q(x_1)), \\ \alpha \rightarrow *(\alpha), & \alpha \rightarrow q(\alpha), & \beta \rightarrow q(\beta) \end{array} \} \\ & *(\alpha) \rightarrow \alpha, & q(\alpha) \rightarrow \alpha, & *(\beta) \rightarrow \beta \end{array} \}$$

- (a) Identify the bottom-up and top-down specific properties of the tree transformations induced by B and T respectively.
- (b) Give td-tt  $T_1$  and  $T_2$  and bu-tt  $B_1$  and  $B_2$  such that  $\tau(B)=\tau(T_1)\circ\tau(T_2)$  and  $\tau(T)=\tau(B_1)\circ\tau(B_2)$ .

## Task 22 (Regular tree grammars)

Consider the ranked alphabet  $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$ , the tree  $\xi = \sigma(\sigma(\alpha, \alpha), \sigma(\alpha, \alpha)) \in T_{\Sigma}$ , and the regular tree grammar  $G = (\{S, A\}, \Sigma, S, \{S \to A, S \to \sigma(S, S), A \to \alpha, A \to \sigma(\alpha, S)\})$ .

- (a) Give a derivation and the corresponding derivation tree of  $\xi$  in G. How many derivation trees of  $\xi$  do exist in G?
- (b) Give an RTG H and a tree  $\zeta$  such that  $\zeta$  has infinitely many derivations in H.
- (c) Give an RTG G' such that G' is in normal form and L(G') = L(G). Give a derivation tree of  $\mathcal{E}$  in G'.
- (d) Prove by construction that for every RTG G there exists an RTG G' in normal form such that L(G') = L(G).