Formale Übersetzungsmodelle

Task 14 ($\mathrm{BOT} \circ \mathrm{HOM} \subseteq \mathrm{BOT}$*)*

Let $B = (Q, \Sigma, \Delta, F, R)$ be a bu-tt and $H = (\{*\}, \Delta, \Omega, \{*\}, R_H)$ a tree homomorphism. Also let $X_{\max} = \{x_i \mid i \in [\max \mathrm{rank}(\Sigma)]\}$. Define the bottom-up tree homomorphism $H' = (\{*\}, \Delta, \Omega \cup X_{\max}, \{*\}, R_H)$ and the bottom-up tree transducer $\hat{B} = (Q, \Sigma, \Omega, F, \hat{R})$ where

$$\begin{aligned} &\sigma(q_1(x_1), ..., q_k(x_k)) \to q(u') \in R \wedge u'[*(x_1), ..., *(x_k)] \Rightarrow^*_{H'} *(t') \\ \iff &\sigma(q_1(x_1), ..., q_k(x_k)) \to q(t') \in \hat{R}\ . \end{aligned}$$

Show that for every $s \in T_\Sigma$, $q \in Q$, and $t \in T_\Delta$ the following equivalence holds:

$$s \Rightarrow^*_{\hat{B}} q(t) \iff \exists u \in T_\Delta \colon s \Rightarrow^*_B q(u) \wedge u \Rightarrow^*_H *(t)\ .$$

Task 15 (Bimorphism characterization of BOT*)*

Recall the decomposition result $\mathrm{BOT} \subseteq \mathrm{REL} \circ \mathrm{FTA} \circ \mathrm{HOM}$. For every bottom-up tree transducer B we can construct a bottom-up tree relabeling B_1, a bottom-up finite state tree automaton B_2, and a bottom-up tree homomorphism B_3 such that $\tau(B) = \tau(B_1) \circ \tau(B_2) \circ \tau(B_3)$.

(a) Show that $(\tau(B_1))^{-1} \in \mathrm{HOM}$.

(b) Give a bimorphism characterization of BOT, i.e., give a formal definition of (the syntax of) bimorphisms, and its induced tree transformation. Show that the class of tree transformations induced by bimorphisms subsumes BOT.

(c) Consider the bu-tt $B = (\{*, q, q_f\}, \Sigma, \Sigma, \{q_f\}, R)$ where $\Sigma = \{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \sigma^{(2)}\}$ and

$$\begin{aligned} R = \{\ &\alpha \to q(\alpha), \quad \alpha \to *(\alpha), \quad \beta \to *(\beta), \quad \gamma(q(x_1)) \to q(\gamma(x_1)), \quad \gamma(*(x_1)) \to *(\gamma(x_1)), \\ &\sigma(*(x_1), q(x_2)) \to q_f(x_1), \quad \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2))\ \} \end{aligned}$$

Give a bimorphism $\mathcal{B} = (A, \varphi, \psi)$ such that $\tau(B) = \tau(\mathcal{B})$.
Give a derivation of $\xi = \sigma(\gamma(\beta), \gamma(\alpha))$ in B.
Give a tree $\zeta \in T_\Omega$ such that $\zeta \in L(A)$ and $\varphi(\zeta) = \xi$.

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