Formale Übersetzungsmodelle

Task 1 (Ranked alphabets and trees)

Consider the following trees:



- (a) Give $\operatorname{height}(\xi_i)$, $\operatorname{size}(\xi_i)$, $\operatorname{pos}(\xi_i)$, $\operatorname{sub}(\xi_i)$ for $i \in \{1, 2\}$.
- (b) Define minimal ranked alphabets Δ_1 and Δ_2 such that $\xi_1 \in T_{\Delta_1}$ and $\xi_2 \in T_{\Delta_2}$.
- (c) Extend the intersection, union, and subset relation to ranked alphabets.
- (d) Prove or refute: There is a ranked alphabet Γ such that $\xi_1, \xi_2 \in T_{\Gamma}$.

Task 2 (Definition by structural induction)

Let $\xi \in T_{\Sigma}$, $w \in \text{pos}(\xi)$, $\zeta \in T_{\Sigma}(X_k)$, and $\zeta'_1, ..., \zeta'_k \in T_{\Sigma}(A)$. Define the following characteristics of ξ and ζ by structural induction:

- (a) $\xi(w)$, the label of ξ at position w,
- (b) $\xi|_w$, the subtree of ξ at position w,
- (c) $\xi[\zeta]_w$, the tree obtained by substituting the subtree of ξ at position w with ζ ,
- (d) yield(ξ), the sequence of leaves of ξ from left to right,
- (e) $\zeta[\zeta'_1,...,\zeta'_k]$, the tree obtained from ζ by substituting x_i by ζ'_i for every $i \in \{1,...,k\}$.

Task 3 (Proof by structural induction)

Let $\xi, \zeta \in T_{\Sigma}(A)$ and $w \in \text{pos}(\xi)$. Prove or refute the following statements:

(a) $\xi(w) = \xi|_w(\varepsilon)$. (b) $(\xi[\zeta]_w)|_w = \zeta$. (c) $|pos(\xi)| = |sub(\xi)|$.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.