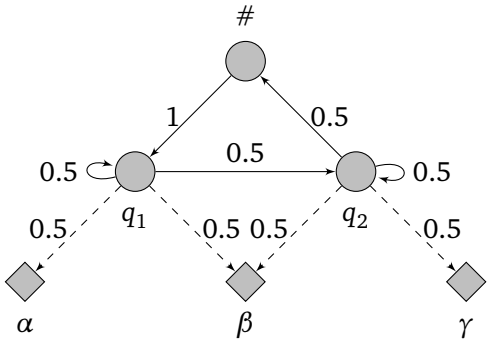
Maschinelles Übersetzen natürlicher Sprachen

9. Übungsblatt

2016-01-07

Aufgabe 1

Consider the hidden Markov model $H = (Q, V, \#, t, e)$ where $Q = \{q_1, q_2\}$, $V = \{\alpha, \beta, \gamma\}$, and t and e are given by the following graphic.



Calculate the following probabilities.

$$P(\mathbb{Q} = q_1 q_1 q_2, \mathbb{V} = \alpha\beta\gamma)$$
$$P(\mathbb{V} = \alpha\beta\gamma)$$
$$P(\mathbb{V} = \alpha\beta^n\gamma) \text{ where } n \in \mathbb{N}$$
$$P(\mathbb{Q} = q_1 q_1 q_2)$$
$$P(\mathbb{V}_2 = \beta)$$

Aufgabe 2

Let $n \in \mathbb{N}$, and let $\mathbb{W}$, $\mathbb{X}$, $\mathbb{Y}$, and $\mathbb{Z}$ be random variables. Show the following implication. If

$$P(\mathbb{W} = w \mid \mathbb{X} = x, \mathbb{Y} = y, \mathbb{Z} = z) = P(\mathbb{W} = w \mid \mathbb{Z} = z) \quad \text{for every } w, x, y, z,$$

then

$$P(\mathbb{W} = w \mid \mathbb{Y} = y, \mathbb{Z} = z) = P(\mathbb{W} = w \mid \mathbb{Z} = z) \quad \text{for every } w, y, z.$$

Aufgabe 3

Let $H = (Q, V, \#, t, e)$ be a hidden Markov model and $v_1 \dots v_n \in V^+$ for some $n \geq 1$.

1. Derive the backward algorithm from the formal definition of HMM by giving a closed and a recursive definition of $S(i, q)$ where $1 \leq i \leq n$ and $q \in Q$. Show that
$$S(i, q) = P(\mathbb{V}_{i+1} = v_{i+1}, \dots, \mathbb{V}_n = v_n, \mathbb{L} = n \mid \mathbb{Q}_i = q).$$
2. Derive the forward algorithm from the formal definition of HMM by giving a closed and a recursive definition of $T(i, q)$ where $1 \leq i \leq n$ and $q \in Q$. Show that
$$T(i, q) = P(\mathbb{V}_1 = v_1, \dots, \mathbb{V}_i = v_i \mid \mathbb{Q}_i = q).$$