Formale Baumsprachen

Task 25 (weighted tree automata)

(a) Let Σ be a ranked alphabet an $e \in \Sigma^{(0)}$. Give a weighted tree automaton $\mathcal{A}$ over a suitable semiring such that $r_{\mathcal{A}}(\xi) = \{\text{yield}(\xi)\}$ for every $\xi \in \mathrm{T}_\Sigma$.

(b) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet and $S = (\mathcal{P}(\mathbb{N}^*), \cup, \circ^R, \emptyset, \{\varepsilon\})$ be a semiring where $U \circ^R V = \{vu \mid u \in U, v \in V\}$ for every $U, V \in \mathcal{P}(\mathbb{N}^*)$. Give an S-weighted tree automaton $\mathcal{B}$ such that $r_{\mathcal{B}}(\xi) = \text{pos}(\xi)$ for every $\xi \in \mathrm{T}_\Sigma$.

Task 26 (analysis of weighted tree automata)

For each of the following weighted tree automata, determine the tree series it induces.

(a) Let $\mathcal{A} = (Q, \Sigma, S, \delta, F)$ be an S-weighted tree automaton where $Q = \{q_1, q_2\}$, $\Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{s^{(0)} \mid s \in S\}$, $(S, +, \cdot, 0, 1)$ is a finite semiring, $F = 1.q_1$, and

$$\begin{aligned} \delta_s &= s.(\varepsilon, q_1) + 1.(\varepsilon, q_2) && \text{for every } s \in S \\ \delta_\oplus &= 1.(q_1 q_2, q_1) + 1.(q_2 q_1, q_1) + 1.(q_2 q_2, q_2) \\ \delta_\odot &= 1.(q_1 q_1, q_1) + 1.(q_2 q_2, q_2) \end{aligned}$$

(b) Let $\mathcal{B} = (P, \Delta, \mathbb{Z}, \tau, G)$ be an $\mathbb{Z}$-weighted tree automaton where $P = \{a, b, q, f\}$, $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$, $(\mathbb{Z}, +, \cdot, 0, 1)$ is the usual ring of integers, $G = 1.f$, and

$$\begin{aligned} \tau_\alpha &= -1.(\varepsilon, a) + 1.(\varepsilon, q) \\ \tau_\beta &= 1.(\varepsilon, a) + 1.(\varepsilon, q) \\ \tau_\gamma &= 1.(a, a) + 1.(b, b) + 1.(q, q) \\ \tau_\sigma &= 1.(aq, a) + 1.(qa, a) + 1.(bq, b) + 1.(qb, b) + 1.(aq, f) + 1.(qb, f) + 1.(qq, q) \end{aligned}$$

Task 27 (constructions on weighted tree automata)

(a) **Product of a recognizable tree series with a scalar.** Let $\mathcal{A} = (Q, \Sigma, S, \delta, F)$ be an S-weighted tree automaton such that S is commutative (i.e. the multiplication of S is commutative). Let $s \in S$. Construct an S-weighted tree automaton $\mathcal{A}'$ such that $r_{\mathcal{A}'} = s \cdot r_{\mathcal{A}}$.

(b) **Sum of recognizable tree series.** Let $\mathcal{B}_1 = (P_1, \Delta, R, \delta_1, G_1)$ and $\mathcal{B}_2 = (P_2, \Delta, R, \delta_2, G_2)$ be R-weighted tree automata. Construct an R-weighted tree automaton $\mathcal{B}'$ such that $r_{\mathcal{B}'} = r_{\mathcal{B}_1} + r_{\mathcal{B}_2}$.

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