Formale Baumsprachen

Task 25 (weighted tree automata)

(a) Let \( \Sigma \) be a ranked alphabet an \( e \in \Sigma^{(0)} \). Give a weighted tree automaton \( \mathcal{A} \) over a suitable semiring such that \( r_{\mathcal{A}}(\xi) = \{\text{yield}(\xi)\} \) for every \( \xi \in T_{\Sigma} \).

(b) Let \( \Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\} \) be a ranked alphabet and \( S = (\mathcal{P}(\mathbb{N}^*), \cup, \circ^R, \emptyset, \{\varepsilon\}) \) be a semiring where \( U \circ^R V = \{uv \mid u \in U, v \in V\} \) for every \( U, V \in \mathcal{P}(\mathbb{N}^*) \). Give an \( S \)-weighted tree automaton \( \mathcal{B} \) such that \( r_{\mathcal{B}}(\xi) = \text{pos}(\xi) \) for every \( \xi \in T_{\Sigma} \).

Task 26 (analysis of weighted tree automata)

For each of the following weighted tree automata, determine the tree series it induces.

(a) Let \( \mathcal{A} = (Q, \Sigma, S, \delta, F) \) be an \( S \)-weighted tree automaton where \( Q = \{q_1, q_2\} \), \( \Sigma = \{\oplus^{(2)}, \odot^{(2)}\} \cup \{s^{(0)} \mid s \in S\} \), \((S, +, \cdot, 0, 1)\) is a finite semiring, \( F = 1.q_1 \), and
\[
\delta_s = s.(\varepsilon, q_1) + 1.(\varepsilon, q_2) \quad \text{for every } s \in S \\
\delta_{\odot} = 1.(q_1q_2, q_1) + 1.(q_2q_1, q_1) + 1.(q_2q_2, q_2) \\
\delta_{\oplus} = 1.(q_1q_1, q_1) + 1.(q_2q_2, q_2)
\]

(b) Let \( \mathcal{B} = (P, \Delta, Z, \tau, G) \) be an \( Z \)-weighted tree automaton where \( P = \{a, b, q, f\} \), \( \Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\} \), \((Z, +, \cdot, 0, 1)\) is the usual ring of integers, \( G = 1.f \), and
\[
\tau_a = -1.(\varepsilon, a) + 1.(\varepsilon, q) \\
\tau_b = 1.(\varepsilon, a) + 1.(\varepsilon, q) \\
\tau_q = 1.(a, a) + 1.(b, b) + 1.(q, q) \\
\tau_f = 1.(aq, a) + 1.(qa, a) + 1.(bq, b) + 1.(qb, b) + 1.(aq, f) + 1.(qb, f) + 1.(qq, q)
\]

Task 27 (constructions on weighted tree automata)

(a) Product of a recognizable tree series with a scalar. Let \( \mathcal{A} = (Q, \Sigma, S, \delta, F) \) be an \( S \)-weighted tree automaton such that \( S \) is commutative (i.e. the multiplication of \( S \) is commutative). Let \( s \in S \). Construct an \( S \)-weighted tree automaton \( \mathcal{A}' \) such that \( r_{\mathcal{A}'} = s \cdot r_{\mathcal{A}} \).

(b) Sum of recognizable tree series. Let \( \mathcal{B}_1 = (P_1, \Delta, R, \delta_1, G_1) \) and \( \mathcal{B}_2 = (P_2, \Delta, R, \delta_2, G_2) \) be \( R \)-weighted tree automata. Construct an \( R \)-weighted tree automaton \( \mathcal{B}' \) such that \( r_{\mathcal{B}'} = r_{\mathcal{B}_1} + r_{\mathcal{B}_2} \).